# NOTE ON A HYPERDETERMINANT IDENTITY.

By *Professor Cayley.*

THE following is in effect a well-known theorem; but I am not sure whether it has been stated in a form at once so general and so precise.

If $\quad \Omega = (*)\,(x_1, y_1)^A\,(x_2, y_2)^B\,(x_3, y_3)^C\,(x_4, y_4)^D \ldots$

be a function separately homogeneous, and of the degrees $A, B, C, D, \ldots$ in the sets of variables $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), \ldots$ respectively; and if

$$\overline{12} = \xi_1\eta_2 - \xi_2\eta_1, = \partial_{x_1}\partial_{y} - \partial_{x_2}\partial_{y_1}, \text{ \&c.},$$

then $$(A\,\overline{23} + B\,\overline{31} + C\,\overline{12})\,\Omega = 0,$$

when the variables $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), \ldots$, or only the variables $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are therein severally replaced by $(x, y)$.

In fact we have

$$A\Omega = (x_1\xi_1 + y_1\eta_1)\,\Omega,\ \ B\Omega = (x_2\xi_2 + y_2\eta_2),\ \ C\Omega = (x_3\xi_3 + y_3\eta_3)\,\Omega\,;$$

thus the expression is

$$= \{(x_1\xi_1 + y_1\eta_1)\,\overline{23} + (y_2\xi_2 + y_2\eta_2)\,\overline{31} + (x_3\xi_3 + y_3\eta_3)\,\overline{12}\}\,\Omega,$$

and if we herein replace the $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ *in so far as they appear explicitly* by $(x, y)$, the expression becomes

$$= \{(x\xi_1 + y\eta_1)\,\overline{23} + (x\xi_2 + y\eta_2)\,\overline{31} + (x\xi_3 + y\eta_3)\,\overline{12}\}\,\Omega,$$

where the factor in { }, substituting for $\overline{23}, \overline{31}, \overline{12}$ their values $\xi_2\eta_3 - \xi_3\eta_2,\ \xi_3\eta_1 - \xi_1\eta_3,\ \xi_1\eta_2 - \xi_2\eta_1$ becomes identically $= 0$. The value of the expression is thus $= 0$, and of course it remains $= 0$, when consequently upon the foregoing change, $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ each into $(x, y)$, we also change $(\xi_1, \eta_1), (\xi_2, \eta_2), (\xi_3, \eta_3)$ each into $(\xi, \eta)$; and if we also change $(x_4, y_4)$, &c., into $(x, y)$.

$\Omega$ may, it is clear, denote the covariant symbol

$$\Omega = \overline{23}^\alpha\,\overline{31}^\beta\,\overline{12}^\gamma\,\overline{14}^\delta\,\overline{24}^\epsilon\,\overline{34}^\zeta \ldots U_1 V_2 W_3 T_4 \ldots,$$

where $U, V, W, T, \ldots$ denote quantics

$$(a, \ldots \between x, y)^m, (a', \ldots \between x, y)^n, \ldots$$

of the degrees $m, n, p, q, \ldots$ respectively, and $U_1$, $V_2$, &c., are the corresponding functions $(a, \ldots \between x_1, y_1)^m$, $(a', \ldots \between x_2, y_2)^n$, &c., the values of $A$, $B$, $C$ being here

$$A = m - \beta - \gamma - \delta - \ldots,$$

$$B = n - \gamma - \alpha - \varepsilon - \ldots,$$

$$C = p - \alpha - \beta - \zeta - \ldots;$$

the theorem expresses that the covariants $\overline{12}\Omega$, $\overline{23}\Omega$, $\overline{31}\Omega$, are linearly connected together; or, writing it in the form $(A\,\overline{12} - C\,\overline{13} + B\,\overline{23})\,\Omega = 0$, we have the proper linear combination $A\,\overline{12}\Omega - C\,\overline{13}\Omega$ of the two covariants $\overline{12}\Omega$ and $\overline{13}\Omega$, equal to $-B\,\overline{23}\Omega$, a determinate multiple of $\overline{23}\Omega$. Speaking roughly, we say that the *difference* of the covariants $\overline{12}\Omega$ and $\overline{13}\Omega$ is equal to $\overline{23}\Omega$.

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# ON THE NONEXISTENCE OF A SPECIAL GROUP OF POINTS.

By Prof. *Cayley*.

It is well known that, taking in a plane any eight points, every cubic through these passes through a determinate ninth point: it is interesting to show that there is no system of seven points such that every cubic through these passes through a determinate eighth point.

Assuming such a system: first, no three of the points can be in a line, for, if they were, then among the cubics through the seven points we have the line through the three points and an arbitrary conic through the remaining four points, and these composite cubics have no common eighth point of intersection.

Secondly, no six of the points can be on a conic, for, if they were, then among the cubics through the seven points we have the conic through the six points and an arbitrary line through the remaining point, and these composite cubics have no common eighth point of intersection.

Taking now the points to be 1, 2, 3, 4, 5, 6, 7; among the cubics through these we have the composite cubics $(A, P)$, $(B, Q)$, $(C, R)$, where $A$, $B$, $C$ are the lines 67, 75, 56, and $P$, $Q$, $R$ the conics 12345, 12346, 12347 respectively; by what precedes, the points 5, 6, 7 do not lie on a line, and the points (6, 7), (7, 5) and (5, 6) neither of them lie on the conics $P$, $Q$, $R$ respectively.