14.

LETTER ON PROFESSOR GALBRAITH'S CONSTRUCTION FOR THE RANGE OF PROJECTILES.

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

Professor Galbraith's geometrical construction for finding the elevations of a projectile corresponding to any given velocity and given range in a plane, horizontal or sloping, is truly elegant, and, if new, constitutes a real acquisition to the subject. It might be worth while for its accomplished author to see if some analogous construction can be found extending to the more general case where the field is a portion of a circle. I need hardly add that the *isoscelism* referred to is, except for some extreme suppositions (impossible to occur in practice), absolutely independent of the form of the field.

As well-constructed names are, in fact, condensed lessons, lending an aid to the memory and imagination, of which modern mathematicians are only beginning to appreciate the importance, I suggest the following designations.

The point of *projection* and point of *impact* speak for themselves; the point vertically over the point of impact in the direction of projection may be called the point of *aim*. The line joining the point of aim and the point of impact is the *drop* or *fall*; the line joining the point of projection and the point of impact may be called the *excursion*; and that joining the point of projection and the point of aim, the *length of aim*.

A vertical section of the ground (plane or curved) through the axis of the gun may be called the *field*. We may then say, that, for the maximum range, the fall is always equal to the excursion, whatever the form of the field; and that in general the locus of the point of aim, for a rectilinear field, when the point of the projection and the velocity are given, is a circle to which, in the

case of the angle of best elevation, the line of fall is of course a tangent. It would not be surprising if a good deal of elegant geometry (like ivy twining round an old wall) should hereafter associate itself with Mr Galbraith's "circle of aim": *à propos* of projectiles, it is not unworthy of observation, that the velocities at any two points P and Q of the parabolic path are as the lines PT, QT which the tangents at P and Q mutually cut off from one another, a remark which of course is easily seen to extend itself to the case of an elliptic orbit with the force in the centre.

Ever, Gentlemen,

Your faithful friend and reader,

J. J. Sylvester.

Woolwich Common,
July 3, 1856.

P.S.   The value of Mr Galbraith's method consists simply in the *act of conception* of the locus of the point of aim; it was scarcely worth while (at this time of day) to append a synthetical proof of so simple a proposition, which may be got at immediately by calling the length of aim ρ, its inclination to the vertical, θ, and that of the field to the vertical, i; when by similar triangles (if H denote the quantity $\dfrac{2v^2}{g}$, and η the vertical distance of the point of projection from the field) we obtain the equation

$$\frac{\frac{\rho^2}{H} - \eta}{\rho} = \frac{\sin(i - \theta)}{\sin i},$$

or

$$\rho^2 - H \frac{\sin(i - \theta)}{\sin i} \rho - H\eta = 0;$$

which obviously corresponds to the circle of Professor Galbraith. I imagine this circle has been long known for the case of the point of projection being in the field, but it may have escaped notice for the more general case. The equality between the *fall* and the *excursion* for the angle of maximum range subsists, not merely for a rectilinear or curved section, but for the ground itself (whatever its form of surface) when the gun is supposed to admit of being laid to any angle, as well as at any elevation.