82.

ON A THEOREM CONCERNING DISCRIMINANTS.

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LET $F(a, b, c, d) = a^2d^2 + 4ac^3 + 4db^3 - 3b^2c^2 - 6abcd$, and let a, b, c, d be four quantities all greater than zero, which make this function vanish.

(1) The cubic equation in x, $F(a, x, c, d) = 0$, will have two positive roots (b, b_1); so $F(a, b_1, x, d)$ will have two such roots (c, c_1), $F(a, x, c_1, d)$ two such (b_1, b_2), $F(a, b_2, x, d)$ two such (c_1, c_2), and so on *ad infinitum*; we may thus generate the infinite series $b_1c_1b_2c_2$.......

Similarly, beginning with the equation $F(a, b, x, d)$, and proceeding as above, we shall obtain a similar series, c', b', c'', b'' ...; and combining the two together, and with the initial quantities b, c, we obtain a series proceeding to infinity in both directions ...... $b'' c'' b' c' b\, c\, b_1\, c_1\, b_2\, c_2$ ....

(2) The four quantities

$$\frac{\delta F}{\delta a}, \frac{\delta F}{\delta b}, \frac{\delta F}{\delta c}, \frac{\delta F}{\delta d},$$

where F represents $F(a, b, c, d)$, will present one or the other of the three following successions of sign,

$$\begin{array}{cccc} + & - & + & - \\ - & + & - & + \\ 0 & 0 & 0 & 0 \end{array}$$

(3) When the last is the case, that is, when the differential derivatives all vanish, the quantities b, c remain stationary in the above double infinite series; in the two other cases, the b quantities and c quantities *continually* increase in one direction and *continually* decrease in the other, the increase taking place in that direction in which we must read the successions of sign of the derivatives of F so as to begin with passing from plus to minus.

(4) To the increase of b and c there is no limit, but to the decrease of each there is a limit, namely $a^{\frac{2}{3}}\, d^{\frac{1}{3}}$ and $a^{\frac{1}{3}}\, d^{\frac{2}{3}}$ are the limits towards which the b and the c terms respectively converge.

I conclude with remarking that the above theorem is only a particular illustration, and the most simple that can be given, of a very wide theory relating to discriminants of all orders which springs as an immediate consequence from the principles involved in the theory of variation of algebraical forms referred to in the note which I had recently the honour of laying before the Society.