86.

ON AN ADDITION TO POINSOT'S ELLIPSOIDAL MODE OF REPRESENTING THE MOTION OF A RIGID BODY TURNING FREELY ROUND A FIXED POINT, WHEREBY THE TIME MAY BE MADE TO REGISTER ITSELF MECHANICALLY.

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THE author of this communication alluded to the well-known geometrical representation invented by Poinsot for exhibiting the motion of a rigid body acted on by no forces, through the medium of an ellipsoid, whose principal moments of inertia are equal to those of the rotating body, and which is supposed to roll without swinging upon a fixed plane, its centre remaining fixed. He pointed out, as an imperfection in this mode of representation, that whilst it exhibits the geometrical path of the body, it gives no image to the mind of the time in which any portion of such path is performed; the velocity of rotation in any position, it is true, is in a manner represented proportionally by the length of the radius vector drawn from the fixed centre to the point of contact of the ellipsoid with the fixed plane; but of the time, there is no indication afforded other than what can be inferred analytically from this law of velocity by means of an integration. The author explained how this imperfection could be remedied and the time put distinctly in evidence, and conceived as registering itself on a dial plate as the ellipsoid continues to roll. For this purpose, the part of the ellipsoid remote from the fixed plane may be conceived to be pared away until its form becomes that of a portion of an ellipsoid confocal with the surface in contact with the fixed plane. This confocal part of the surface may be conceived to be brought into contact with a fixed plate parallel to the fixed plane, and capable of only one motion, namely, of revolution round an axis, which, if produced, would pass through the fixed centre, and perpendicular to the fixed plane. It was explained how the ellipsoid in the act of rolling upon the last-named plane would roll upon the plate parallel to it, and at the same time by the friction drive the plate round its axis: this angular rotation could be measured upon

a fixed dial-face immediately above the plate, and the amount of it would be always *precisely proportional* to the time that would be occupied by the body which the ellipsoid represents, if perfectly free, in passing from any given position to any other; that is to say, the representative ellipsoid passing from a position A to a position B, and in the act of passing from A to B, driving the rotating plate through the angle U, U would measure the time in which the free body could pass from A to B. If a^2, b^2, c^2 be the semi-axes squared of Poinsot's ellipsoid, $a^2-\lambda$, $b^2-\lambda$, $c^2-\lambda$ of the confocal one above described, L the initial impulsive moment of the free body, the time between the position A and position B will be measured by $\frac{U}{L\lambda}$; so that the initial impulse being supposed given, the relation between the time and the magnitude of the divisions in the dial-plate is invariable; and thus by supposing the fixed plane and the parallel plate to admit, by a preliminary adjustment, of being set at any required distance from one another, the same instrument would continue to measure on the same scale the time of the free body passing from any initial position whatever to any other position into which it could turn.