97.

ON THE SUCCESSIVE INVOLUTES TO A CIRCLE.

FROM the first involute of a circle we may derive a family of parallel curves forming the second involutes of the circle; from each of these again families, the totality of which will form the third involutes, and so on continually.

The author had been led by circumstances to study the arco-radial or semi-intrinsic equation of these curves, and had arrived at certain conclusions concerning its form which subsequent investigations have verified: it turns out that the general equation between the arc s and radius vector r of the general involute of the nth degree will be found by taking F, any rational integer function of x of the nth degree, and eliminating x between the equations

$$r^2 = F^2 + \left(\frac{dF}{dx}\right)^2, \quad s = \int dx F + \frac{dF}{dx}.$$

It follows, as the author had surmised, that the general arco-radial equation for the involute of the nth order when n exceeds unity, is of the degree $(n+1)$ in r^2 and $2n$ in s. Of course, in the case of n equal to unity, the degrees sink to 1 in r^2 and 1 in s. The second involute formed by unwrapping from the cusp of the first may be termed the natural second involute, but is not the most simple of the family; this, which is at the normal distance of half the radius externally from the one last named, is of the third degree in r and the second in s. It may be derived from the curve which a fixed point in a wall at half the length of the radius of a wheel from the ground marks in the wheel as it rolls along the face of the wall by doubling the vectorial angles and taking the squares of the radii vectores. From the arco-radial it is easy to pass to the general polar equation to the n-ary involute; the equation between p, the perpendicular from the centre and q the polar subtangent, is also very easily obtained, being, in fact, no other than the result of eliminating between $Fx = p$, $F'x = q$, Fx being any quantic in x of the nth degree, so that this equation will be of the $(n-1)$th order in q, and the nth order in p.

In the *Philosophical Magazine* for October and December, and in the *Proceedings of the Mathematical Society of London* [below], will be found further developments of the theory of these circular involutes, which it is proposed to term Cyclodes.