100.

PRESIDENTIAL ADDRESS TO SECTION 'A' OF THE BRITISH ASSOCIATION.

LADIES AND GENTLEMEN,—

A few days ago I noticed in a shop window the photograph of a Royal mother and child, which seemed to me a very beautiful group; on scanning it more closely, I discovered that the faces were ordinary, or, at all events, not much above the average, and that the charm arose entirely from the natural action and expression of the mother stooping over and kissing her child which she held in her lap; and I remarked to myself that the homeliest features would become beautiful when lit up by the rays of the soul—like the sun "gilding pale streams with heavenly alchemy." By analogy, the thought struck me that if a man would speak naturally and as he felt on any subject of his predilection, he might hope to awaken a sympathetic interest in the minds of his hearers; and, in illustration of this, I remembered witnessing how the writer of a well-known article in the *Quarterly Review* so magnetized his audience at the Royal Institution by his evident enthusiasm that, when the lecture was over and the applause had subsided, some ladies came up to me and implored me to tell them what they should do to get up the Talmud; for that was what the lecture had been about.

Now, as I believe that even Mathematics are not much more repugnant than the Talmud to the common apprehension of mankind, and I really love my subject, I shall not quite despair of rousing and retaining your attention for a short time if I proceed to read (as, for greater assurance against breaking down, I shall beg your permission to do) from the pages I hold in my hand.

It is not without a feeling of surprise and trepidation at my own temerity that I find myself in the position of one about to address this numerous and distinguished assembly. When informed that the Council of the British Association had it in contemplation to recommend me to the General Committee to fill the office of President of the Mathematical and Physical

[* The Address was also reprinted by the Author in a volume issued by Longmans, Green and Co., London, 1870, of which the earlier portion deals with the Laws of Verse. Some additional notes to the Address there given are reproduced at the end of this volume.]

Section, the intimation was accompanied with the tranquilizing assurance that it would rest with myself to deliver or withhold an address as I might think fit, and that I should be only following in the footsteps of many of the most distinguished of my predecessors were I to resolve on the latter course.

Until the last few days I had made up my mind to avail myself of this option, by proceeding at once to the business before us without troubling you to listen to any address, swayed thereto partly by a consciousness of the very limited extent of my oratorical powers, partly by a disinclination, in the midst of various pressing private and official occupations, to undertake a kind of work new to one more used to thinking than to speaking (to making mathematics than to talking about them), and partly and more especially by a feeling of my inadequacy to satisfy the expectations that would be raised in the minds of those who had enjoyed the privilege of hearing or reading the allocution (which fills me with admiration and dismay) of my gifted predecessor, Dr Tyndall, a man in whom eloquence and philosophy seem to be inborn, whom Science and Poetry woo with an equal spell*, and whose ideas have a faculty of arranging themselves in forms of order and beauty as spontaneously and unfailingly as those crystalline solutions from which, in a striking passage of his address, he drew so vivid and instructive an illustration.

From this lotus-eater's dream of fancied security and repose I was rudely awakened by receiving from the Editor of an old-established journal in this city a note containing a polite but peremptory request that I should, at my earliest convenience, favour him with a "copy of the address I proposed to deliver at the forthcoming Meeting." To this invitation, my first impulse was to respond very much in the same way as did the "Needy knife-grinder" of the *Antijacobin,* when summoned to recount the story of his wrongs to his republican sympathizer, "Story, God bless you, I have none to tell, Sir!" "Address, Mr Editor, I have none to deliver."

I have found, however, that increase of appetite still grows with what it feeds on, that those who were present at the opening of the Section last year,

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* So it is said of Jacobi, that he attracted the particular attention and friendship of Böckh, the director of the philological seminary at Berlin, by the zeal and talent he displayed for philology, and only at the end of two years' study at the University, and after a severe mental struggle, was able to make his final choice in favour of mathematics. The relation between these two sciences is not perhaps so remote as may at first sight appear, and indeed it has often struck me that metamorphosis runs like a golden thread through the most diverse branches of modern intellectual culture, and forms a natural link of connexion between subjects in their aims so unlike as Grammar, Ethnology, Rational Mythology, Chemistry, Botany, Comparative Anatomy, Physiology, Physics, Algebra, Music, all of which, under the modern point of view, may be regarded as having morphology for their common centre. Even singing, I have been told, the advanced German theorists regard as being strictly a development of recitative, and infer therefrom that no essentially new melodic themes can be invented until a social cataclysm, or the civilization of some at present barbaric races, shall have created new necessities of expression and called into activity new forms of impassioned declamation.

and enjoyed my friend Dr Tyndall's melodious utterances, would consider themselves somewhat ill-treated if they were sent away quite empty on the present occasion, and that, failing an address, the Members would feel very much like the guests at a wedding-breakfast where no one was willing or able to propose the health of the bride and bridegroom.

Yielding, therefore, to these considerations and to the advice of some officially connected with the Association, to whose opinions I feel bound to defer, and unwilling also to countenance by my example the too prevailing opinion that mathematical pursuits unfit a person for the discharge of the common duties of life and cut him off from the exercise of Man's highest prerogative, "discourse of reason and faculty of speech divine,"—rather, I say, than favour the notion that we Algebraists (who regard each other as the flower and salt of the earth) are a set of mere calculating-machines endowed with organs of locomotion, or, at best, a sort of poor visionary dumb creatures only capable of communicating by signs and symbols with the outer world, I have resolved to take heart of grace and to say a few words, which I hope to render, if not interesting, at least intelligible, on a subject to which the larger part of my life has been devoted.

The President of the Association, Prof. Stokes, is so eminent alike as a mathematician and physicist, and so distinguished for accuracy and extent of erudition and research, that I felt assured I might safely assume he would, in his Address to the Association at large, take an exhaustive survey, and render a complete account of the recent progress and present condition and prospects of Mathematical and Physical Science. This consideration narrowed very much and brought almost to a point the ground available for me to occupy in this Section; and as I cannot but be aware that it is as a cultivator of pure mathematics (the subject in which my own researches have chiefly, though by no means exclusively lain*) that I have been placed in this Chair, I hope the Section will patiently bear with me in the observations I shall venture to make on the nature of that province of the human reason and its title to the esteem and veneration with which through countless ages it has

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* My first printed paper was on Fresnel's Optical Theory, published in the *Philosophical Magazine*; my latest contribution to the *Philosophical Transactions* is a memoir on the "Rotation of a Free Rigid Body." There is an old adage, "purus mathematicus, purus asinus." On the other hand, I once heard the great Richard Owen say, when we were opposite neighbours in Lincoln's-Inn Fields (doves nestling among hawks), that he would like to see *Homo Mathematicus* constituted into a distinct subclass, thereby suggesting to my mind sensation, perception, reflection, abstraction, as the successive stages or phases of protoplasm on its way to being made perfect in Mathematicised Man. Would it sound too presumptuous to speak of perception as a quintessence of sensation, language (that is, communicable thought) of perception, mathematic of language? We should then have four terms differentiating from inorganic matter and from each other the Vegetable, Animal, Rational, and Supersensual modes of existence.

been and, so long as Man respects the intellectual part of his nature, must ever continue to be regarded*.

It is said of a great party leader and orator in the House of Lords that, when lately requested to make a speech at some religious or charitable (at all events a non-political) meeting, he declined to do so on the ground that he could not speak unless he saw an adversary before him—somebody to attack or reply to. In obedience to a somewhat similar combative instinct, I set to myself the task of considering certain recent utterances of a most distinguished member of this Association, one whom I no less respect for his honesty and public spirit than I admire for his genius and eloquence†, but from whose opinions on a subject which he has not studied I feel constrained to differ. Goethe has said—

> "Verständige Leute kannst du irren sehn
> In Sachen, nämlich, die sie nicht verstehn."

Understanding people you may see erring—in those things, to wit, which they do not understand.

I have no doubt that had my distinguished friend, the probable President-elect of the next Meeting of the Association, applied his uncommon powers of reasoning, induction, comparison, observation, and invention to the study of mathematical science, he would have become as great a mathematician as he is now a biologist; indeed he has given public evidence of his ability to grapple with the practical side of certain mathematical questions; but he has not made a study of mathematical science as such, and the eminence of his position and the weight justly attaching to his name render it only the more imperative that any assertions proceeding from such a quarter, which may appear to me erroneous, or so expressed as to be conducive to error, should not remain unchallenged or be passed over in silence‡.

He says "mathematical training is almost purely deductive. The mathematician starts with a few simple propositions, the proof of which is so obvious that they are called self-evident, and the rest of his work consists of

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* Mr Spottiswoode favoured the Section, in his opening address, with a combined history of the progress of Mathematics and Physics; Dr Tyndall's address was virtually on the limits of Physical Philosophy; the one here in print is an attempted faint adumbration of the nature of Mathematical Science in the abstract. What is wanting (like a fourth sphere resting on three others in contact) to build up the Ideal Pyramid is a discourse on the Relation of the two branches (Mathematics and Physics) to, their action and reaction upon, one another, a magnificent theme with which it is to be hoped some future President of Section A will crown the edifice and make the Tetralogy (symbolizable by $A + A'$, A, A', $A \cdot A'$) complete.

† Although no great lecture-goer, I have heard three lectures in my life which have left a lasting impression as masterpieces on my memory—Clifford on Mind, Huxley on Chalk, Dumas on Faraday.

‡ In his *éloge* of Daubenton, Cuvier remarks, "Les savants jugent toujours comme le vulgaire les ouvrages qui ne sont pas de leur genre."

subtle deductions from them. The teaching of languages, at any rate as ordinarily practised, is of the same general nature—authority and tradition furnish the data, and the mental operations are deductive." It would seem from the above somewhat singularly juxtaposed paragraphs that, according to Prof. Huxley, the business of the mathematical student is from a limited number of propositions (bottled up and labelled ready for future use) to deduce any required result by a process of the same general nature as a student of language employs in declining and conjugating his nouns and verbs—that to make out a mathematical proposition and to construe or parse a sentence are equivalent or identical mental operations. Such an opinion scarcely seems to need serious refutation. The passage is taken from an article in *Macmillan's Magazine* for June last, entitled "Scientific Education—Notes of an After-dinner Speech," and I cannot but think would have been couched in more guarded terms by my distinguished friend had his speech been made *before* dinner instead of *after*.

The notion that mathematical truth rests on the narrow basis of a limited number of elementary propositions from which all others are to be derived by a process of logical inference and verbal deduction, has been stated still more strongly and explicitly by the same eminent writer in an article of even date with the preceding in the *Fortnightly Review*, where we are told that " Mathematics is that study which knows nothing of observation, nothing of experiment, nothing of induction, nothing of causation." I think no statement could have been made more opposite to the undoubted facts of the case, that mathematical analysis is constantly invoking the aid of new principles, new ideas, and new methods, not capable of being defined by any form of words, but springing direct from the inherent powers and activity of the human mind, and from continually renewed introspection of that inner world of thought of which the phenomena are as varied and require as close attention to discern as those of the outer physical world (to which the inner one in each individual man may, I think, be conceived to stand in somewhat the same general relation of correspondence as a shadow to the object from which it is projected, or as the hollow palm of one hand to the closed fist which it grasps of the other), that it is unceasingly calling forth the faculties of observation and comparison, that one of its principal weapons is induction, that it has frequent recourse to experimental trial and verification, and that it affords a boundless scope for the exercise of the highest efforts of imagination and invention.

Lagrange, than whom no greater authority could be quoted, has expressed emphatically his belief in the importance to the mathematician of the faculty of observation; Gauss has called mathematics a science of the eye, and in conformity with this view always paid the most punctilious attention to preserve his text free from typographical errors; the ever to be lamented

Riemann has written a thesis to show that the basis of our conception of space is purely empirical, and our knowledge of its laws the result of observation, that other kinds of space might be conceived to exist subject to laws different from those which govern the actual space in which we are immersed, and that there is no evidence of these laws extending to the ultimate infinitesimal elements of which space is composed. Like his master Gauss, Riemann refuses to accept Kant's doctrine of space and time being forms of intuition, and regards them as possessed of physical and objective reality. I may mention that Baron Sartorius von Waltershausen (a member of this Association) in his biography of Gauss ("Gauss zu Gedächtniss"), published shortly after his death, relates that this great man used to say that he had laid aside several questions which he had treated analytically, and hoped to apply to them geometrical methods in a future state of existence, when his conceptions of space should have become amplified and extended; for as we can conceive beings (like infinitely attenuated bookworms* in an infinitely thin sheet of paper) which possess only the notion of space of two dimensions, so we may imagine beings capable of realising space of four or a greater number of dimensions†. Our Cayley, the central luminary, the Darwin of the English school of mathematicians, started and elaborated at an early age, and with happy consequences, the same bold hypothesis.

Most, if not all, of the great ideas of modern mathematics have had their origin in observation. Take, for instance, the arithmetical theory of forms, of which the foundation was laid in the diophantine theorems of Fermat, left without proof by their author, which resisted all the efforts of the myriad-minded Euler to reduce to demonstration, and only yielded up their cause of being when turned over in the blowpipe flame of Gauss's transcendent genius; or the doctrine of double periodicity, which resulted from the observation by Jacobi of a purely analytical fact of transformation; or Legendre's law of reciprocity; or Sturm's theorem about the roots of equations, which, as he informed me with his own lips, stared him in the face

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* I have read or been told that eye of observer has never lighted on these depredators, living or dead. Nature has gifted me with eyes of exceptional microscopic power, and I can speak with some assurance of having repeatedly seen the creature wriggling on the learned page. On approaching it with breath or finger-nail it stiffens out into the semblance of a streak of dirt, and so eludes detection.

† It is well known to those who have gone into these views that the laws of motion accepted as a fact suffice to prove in a general way that the space we live in is a flat or level space (a "homaloid"), our existence therein being assimilable to the life of the bookworm in an *unrumpled page*: but what if the page should be undergoing a process of gradual bending into a curved form? Mr W. K. Clifford has indulged in some remarkable speculations as to the possibility of our being able to infer, from certain unexplained phenomena of light and magnetism, the fact of our level space of three dimensions being in the act of undergoing in space of four dimensions (space as inconceivable to us as our space to the supposititious bookworm) a distortion analogous to the rumpling of the page to which that creature's powers of direct perception have been postulated to be limited.

in the midst of some mechanical investigations connected with the motion of compound pendulums; or Huyghens' method of continued fractions, characterized by Lagrange as one of the principal discoveries of "that great mathematician, and to which he appears to have been led by the construction of his Planetary Automaton"; or the New Algebra, speaking of which one of my predecessors (Mr Spottiswoode) has said, not without just reason and authority, from this Chair, "that it reaches out and indissolubly connects itself each year with fresh branches of mathematics, that the theory of equations has almost become new through it, algebraic geometry transfigured in its light, that the calculus of variations, molecular physics, and mechanics" (he might, if speaking at the present moment, go on to add the theory of elasticity and the highest developments of the integral calculus) "have all felt its influence."

Now this gigantic outcome of modern analytical thought, itself, too, only the precursor and progenitor of a future still more heaven-reaching theory, which will comprise a complete study of the interoperation, the actions and reactions, of algebraic forms (Analytical Morphology in its absolute sense), how did this originate? In the accidental observation by Eisenstein, some score or more years ago, of a single invariant (the Quadrinvariant of a Binary Quartic) which he met with in the course of certain researches just as accidentally and unexpectedly as M. Du Chaillu might meet a Gorilla in the country of the Fantees, or any one of us in London a White Polar Bear escaped from the Zoological Gardens. Fortunately he pounced down upon his prey and preserved it for the contemplation and study of future mathematicians. It occupies only part of a page in his collected posthumous works. This single result of observation (as well entitled to be so called as the discovery of Globigerinæ in chalk or of the Confoco-ellipsoidal structure of the shells of the Foraminifera), which remained unproductive in the hands of its distinguished author, has served to set in motion a train of thought and to propagate an impulse which have led to a complete revolution in the whole aspect of modern analysis, and whose consequences will continue to be felt until Mathematics are forgotten and British Associations meet no more.

I might go on, were it necessary, piling instance upon instance to prove the paramount importance of the faculty of observation to the process of mathematical discovery*. Were it not unbecoming to dilate on one's personal experience, I could tell a story of almost romantic interest about my

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* Newton's Rule was to all appearance, and according to the more received opinion, obtained inductively by its author. My own reduction of Euler's problem of the Virgins (or rather one slightly more general than this) to the form of a question (or, to speak more exactly, a set of questions) in simple partitions was, strange to say, first obtained by myself inductively, the result communicated to Prof. Cayley, and proved subsequently by each of us independently, and by perfectly distinct methods.

own latest researches in a field where Geometry, Algebra, and the Theory of Numbers melt in a surprising manner into one another, like sunset tints or the colours of the dying dolphin, "the last still loveliest" (a sketch of which has just appeared in the *Proceedings of the London Mathematical Society**), which would very strikingly illustrate how much observation, divination, induction, experimental trial, and verification, causation, too (if that means, as I suppose it must, mounting from phenomena to their reasons or causes of being), have to do with the work of the mathematician. In the face of these facts, which every analyst in this room or out of it can vouch for out of its own knowledge and personal experience, how can it be maintained, in the words of Professor Huxley, who, in this instance, is speaking of the sciences as they are in themselves and without any reference to scholastic discipline, that Mathematics "is that study which knows nothing of observation, nothing of induction, nothing of experiment, nothing of causation"?

I, of course, am not so absurd as to maintain that the habit of observation of external nature will be best or in any degree cultivated by the study of mathematics, at all events as that study is at present conducted; and no one can desire more earnestly than myself to see natural and experimental science introduced into our schools as a primary and indispensable branch of education: I think that that study and mathematical culture should go on hand in hand together, and that they would greatly influence each other for their mutual good. I should rejoice to see mathematics taught with that life and animation which the presence and example of her young and buoyant sister could not fail to impart, short roads preferred to long ones, Euclid honourably shelved or buried "deeper than did ever plummet sound" out of the schoolboy's reach, morphology introduced into the elements of Algebra—projection, correlation, and motion accepted as aids to geometry—the mind of the student quickened and elevated and his faith awakened by early initiation into the ruling ideas of polarity, continuity, infinity, and familiarization with the doctrine of the imaginary and inconceivable.

It is this living interest in the subject which is so wanting in our traditional and mediaeval modes of teaching. In France, Germany, and Italy, everywhere where I have been on the Continent, mind acts direct on mind in a manner unknown to the frozen formality of our academic institutions schools of thought and centres of real intellectual cooperation exist; the relation of master and pupil is acknowledged as a spiritual and a lifelong tie, connecting successive generations of great thinkers with each other in an unbroken chain, just in the same way as we read, in the catalogue of our French Exhibition, or of the Salon at Paris, of this man or that being the pupil of one great painter or sculptor and the master of another. When followed out in this spirit, there is no study in the world which brings into

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* Under the title of "Outline Trace of the Theory of Reducible Cyclodes." [Below, p. 663.]

more harmonious action all the faculties of the mind than the one of which I stand here as the humble representative, there is none other which prepares so many agreeable surprises for its followers, more wonderful than the changes in the transformation-scene of a pantomime, or, like this, seems to raise them, by successive steps of initiation, to higher and higher states of conscious intellectual being.

This accounts, I believe, for the extraordinary longevity of all the greatest masters of the Analytical art, the Dii Majores of the mathematical Pantheon. Leibnitz lived to the age of 70 ; Euler to 76 ; Lagrange to 77 ; Laplace to 78 ; Gauss to 78 ; Plato, the supposed inventor of the conic sections, who made mathematics his study and delight, who called them the handles or aids to philosophy, the medicine of the soul, and is said never to have let a day go by without inventing some new theorems, lived to 82 ; Newton, the crown and glory of his race, to 85 ; Archimedes, the nearest akin, probably, to Newton in genius, was 75, and might have lived on to be 100, for aught we can guess to the contrary, when he was slain by the impatient and ill-mannered sergeant, sent to bring him before the Roman general, in the full vigour of his faculties, and in the very act of working out a problem ; Pythagoras, in whose school, I believe, the word mathematician (used, however, in a somewhat wider than its present sense) originated, the second founder of geometry, the inventor of the matchless theorem which goes by his name, the precognizer of the undoubtedly mis-called Copernican theory, the discoverer of the regular solids and the musical canon, who stands at the very apex of this pyramid of fame, (if we may credit the tradition) after spending 22 years studying in Egypt, and 12 in Babylon, opened school when 56 or 57 years old in Magna Græcia, married a young wife when past 60, and died, carrying on his work with energy unspent to the last, at the age of 99. The mathematician lives long and lives young ; the wings of his soul do not early drop off, nor do its pores become clogged with the earthy particles blown from the dusty highways of vulgar life.

Some people have been found to regard all mathematics, after the 47th proposition of Euclid, as a sort of morbid secretion, to be compared only with the pearl said to be generated in the diseased oyster, or, as I have heard it described, "une excroissance maladive de l'esprit humain." Others find its justification, its "raison d'être," in its being either the torch-bearer leading the way, or the handmaiden holding up the train of Physical Science ; and a very clever writer in a recent magazine article, expresses his doubts whether it is, in itself, a more serious pursuit, or more worthy of interesting an intellectual human being, than the study of chess problems or Chinese puzzles. What is it to us, they say, if the three angles of a triangle are equal to two right angles, or if every even number is, or may be, the sum of two primes, or if every equation of an odd degree must have a real root.

How dull, stale, flat, and unprofitable are such and such like announcements! Much more interesting to read an account of a marriage in high life, or the details of an international boat-race. But this is like judging of architecture from being shown some of the brick and mortar, or even a quarried stone of a public building, or of painting from the colours mixed on the palette, or of music by listening to the thin and screechy sounds produced by a bow passed haphazard over the strings of a violin. The world of ideas which it discloses or illuminates, the contemplation of divine beauty and order which it induces, the harmonious connexion of its parts, the infinite hierarchy and absolute evidence of the truths with which it is concerned, these, and such like, are the surest grounds of the title of mathematics to human regard, and would remain unimpeached and unimpaired were the plan of the universe unrolled like a map at our feet, and the mind of man qualified to take in the whole scheme of creation at a glance.

In conformity with general usage, I have used the word mathematics in the plural; but I think it would be desirable that this form of word should be reserved for the applications of the science, and that we should use mathematic in the singular number to denote the science itself, in the same way as we speak of logic, rhetoric, or (own sister to algebra*) music. Time was when all the parts of the subject were dissevered, when algebra, geometry, and arithmetic either lived apart or kept up cold relations of acquaintance confined to occasional calls upon one another; but that is now at an end; they are drawn together and are constantly becoming more and more intimately related and connected by a thousand fresh ties, and we may confidently look forward to a time when they shall form but one body with one soul. Geometry formerly was the chief borrower from arithmetic and algebra, but it has since repaid its obligations with abundant usury; and if I were asked to name, in one word, the pole-star round which the mathematical firmament revolves, the central idea which pervades as a hidden spirit the whole corpus of mathematical doctrine, I should point to Continuity as contained in our notions of space, and say, it is this, it is this! Space is the *Grand Continuum* from which, as from an inexhaustible reservoir, all the fertilizing ideas of modern analysis are derived; and as Brindley, the engineer, once allowed before a parliamentary committee that, in his opinion, rivers were made to feed navigable canals, I feel almost tempted to say that one principal reason for the existence of space, or at least one principal function which it discharges, is that of feeding mathematical invention. Everybody knows what a wonderful influence geometry has exercised in the hands of

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* I have elsewhere (in my 'Trilogy' [above, p. 419, *footnote*] published in the *Philosophical Transactions*) referred to the close connexion between these two cultures, not merely as having Arithmetic for their common parent, but as similar in their habits and affections. I have called "Music the Algebra of sense, Algebra the Music of the reason; Music the dream, Algebra the waking life,—the soul of each the same!"

42—2

Cauchy, Puiseux, Riemann, and his followers Clebsch, Gordan, and others, over the very form and presentment of the modern calculus, and how it has come to pass that the tracing of curves, which was once to be regarded as a puerile amusement, or at best useful only to the architect or decorator, is now entitled to take rank as a high philosophical exercise, inasmuch as every new curve or surface, or other circumscription of space is capable of being regarded as the embodiment of some specific organized system of continuity*.

The early study of Euclid made me a hater of Geometry, which I hope may plead my excuse if I have shocked the opinions of any in this room (and I know there are some who rank Euclid as second in sacredness to the Bible alone, and as one of the advanced outposts of the British Constitution) by the tone in which I have previously alluded to it as a school-book; and yet, in spite of this repugnance, which had become a second nature in me, whenever I went far enough into any mathematical question, I found I touched, at last, a geometrical bottom : so it was, I may instance, in the purely arithmetical theory of partitions; so, again, in one of my more recent studies, the purely algebraical question of the invariantive criteria of the nature of the roots of an equation of the fifth degree : the first inquiry landed me in a new theory of polyhedra; the latter found its perfect and only possible complete solution in the construction of a surface of the ninth order and the subdivision of its infinite content into three distinct natural regions.

Having thus expressed myself at much greater length than I originally intended on the subject, which, as standing first on the muster-roll of the Association, and as having been so recently and repeatedly arraigned before the bar of public opinion, is entitled to be heard in its defence (if anywhere) in this place,—having endeavoured to show what it is not, what it is, and what it is probably destined to become, I feel that I must enough and more than enough have trespassed on your forbearance, and shall proceed with the regular business of the Meeting.

Before calling upon the authors of the papers contained in the varied bill of intellectual fare which I see before me, I hope to be pardoned if I direct attention to the importance of practising brevity and condensation in the delivery of communications to the Section, not merely as a saving of valuable time, but in order that what is said may be more easily followed and listened to with greater pleasure and advantage. I believe that immense good may be done by the oral interchange and discussion of ideas which takes place in the Sections; but for this to be possible, details and long descriptions should be reserved for printing and reading, and only the general outlines and broad

* M. Camille Jordan's application of Dr Salmon's Eikosi-heptagram to Abelian functions is one of the most recent instances of this reverse action of geometry on analysis. Mr Crofton's admirable apparatus of a reticulation with infinitely fine meshes rotated successively through indefinitely small angles, which he applies to obtaining whole families of definite integrals, is another equally striking example of the same phenomenon.

statements of facts, methods, observations, or inventions brought before us here, such as can be easily followed by persons having a fair average acquaintance with the several subjects treated upon. I understand the rule to be that, with the exception of the author of any paper who may answer questions and reply at the end of the discussion, no member is to address the Section more than once on the same subject, or occupy more than a quarter of an hour in speaking.

In order to get through the business set down in each day's paper, it may sometimes be necessary for me to bring a discussion to an earlier close than might otherwise be desirable, and for that purpose to request the authors of papers, and those who speak upon them, to be brief in their addresses. I have known most able investigators at these Meetings, and especially in this Section, gradually part company with their audience, and at last become so involved in digressions as to lose entirely the thread of their discourse, and seem to forget, like men waking out of sleep, where they were or what they were talking about. In such cases I shall venture to give a gentle pull to the string of the kite before it soars right away out of sight into the region of the clouds. I now call upon Dr Magnus to read his paper and recount to the Section his wondrous story on the Emission, Absorption, and Reflection of Obscure Heat*.

POSTSCRIPT.—The remarks on the use of experimental methods in mathematical investigation led to Dr Jacobi, the eminent physicist of St Petersburg, who was present at the delivery of the address, favouring me with the annexed anecdote relative to his illustrious brother C. G. J. Jacobi.

"En causant un jour avec mon frère défunt sur la necessité de contrôler par des expériences réitérées toute observation, même si elle confirme l'hypothèse, il me raconta avoir découvert un jour une loi très-remarquable de la théorie des nombres, dont il ne douta guère qu'elle fût générale. Cependant par un excès de précaution ou plutôt pour faire le superflu, il voulut substituer un chiffre quelconque réel aux termes généraux, chiffre qu'il choisit au hasard ou, peut-être, par une espèce de divination, car en effet ce chiffre mit sa formule en défaut; tout autre chiffre qu'il essaya en confirma la généralité. Plus tard il réussit à prouver que le chiffre choisi par lui par hasard, appartenait à un système de chiffres qui faisait la seule exception à la règle.

"Ce fait curieux m'est resté dans la mémoire, mais comme il s'est passé il y a plus d'une trentaine d'années, je ne rappelle plus des détails.

"M. H. JACOBI."

"EXETER, 24. *Août*, 1869."

* Curiously enough, and as if symptomatic of the genial warmth of the proceedings in which seven sages from distant lands (Jacobi, Magnus, Newton, Janssen, Morren, Lyman, Neumayer) took frequent part, the opening and concluding papers (each of surpassing interest, and a letting-out of mighty waters) were on Obscure Heat, by Prof. Magnus, and on Stellar Heat, by Mr Huggins.