

MATHEMATICAL
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JAMES JOHNSON

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THE COLLECTED
MATHEMATICAL PAPERS

OF

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THE COLLECTED
MATHEMATICAL PAPERS
OF

JAMES JOSEPH SYLVESTER

F.R.S. F.R.I.C. F.R.S.E.

Professor of Mathematics in the University of London
and in the University of Cambridge
and in the University of Oxford
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and in the University of Stirling
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PREFATORY NOTE.

AMONG the Papers contained in this Volume are, the Author's Lecture on Geometry, delivered before the Gresham Committee (No. 2), the Author's seven lectures on the Partition of Numbers, in outline (No. 26), the long memoir on Newton's Rule (No. 74), and the Presidential Address to the Mathematical and Physical Section of the British Association at Exeter (No. 100). The papers here numbered 87, 88, 89 and 94 were republished together with the title "Nugae Mathematicae," and are found in the British Museum Catalogue under that name.

As in the first Volume, save for obvious errors of algebraical formulae, the papers are reprinted unaltered, cross references, and—in a few cases—indications of correction, being enclosed in square brackets.

A Table of Contents is prefixed, a General Index being deferred to the last Volume.

H. F. BAKER.

ST JOHN'S COLLEGE, CAMBRIDGE.

2 March 1908.

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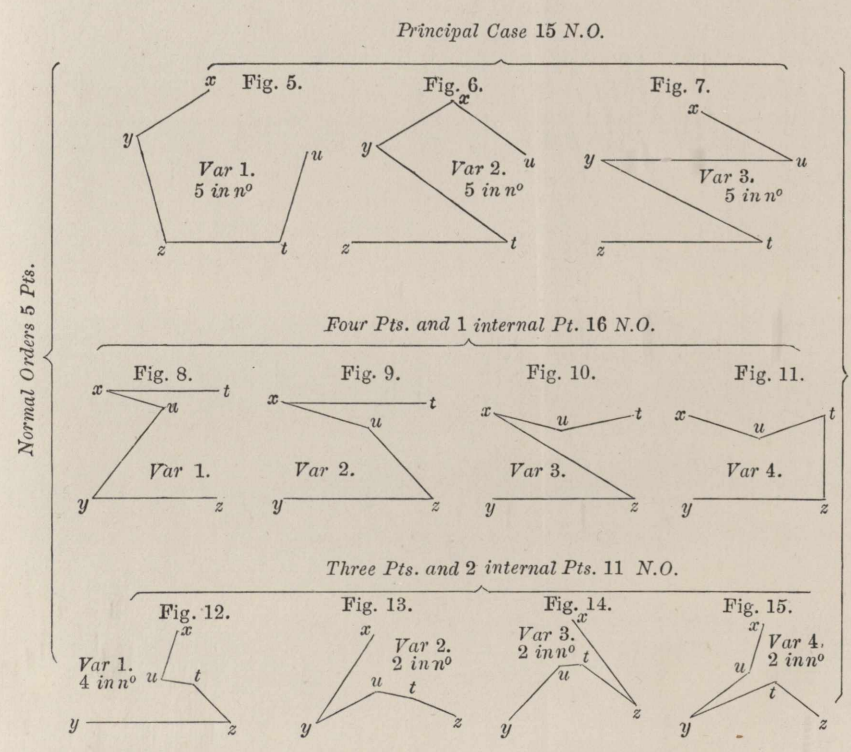
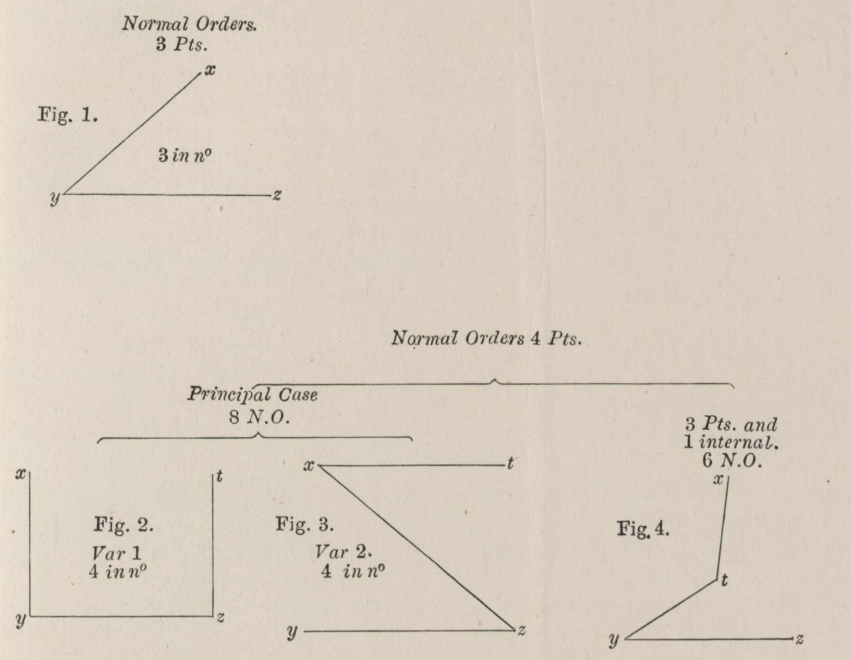
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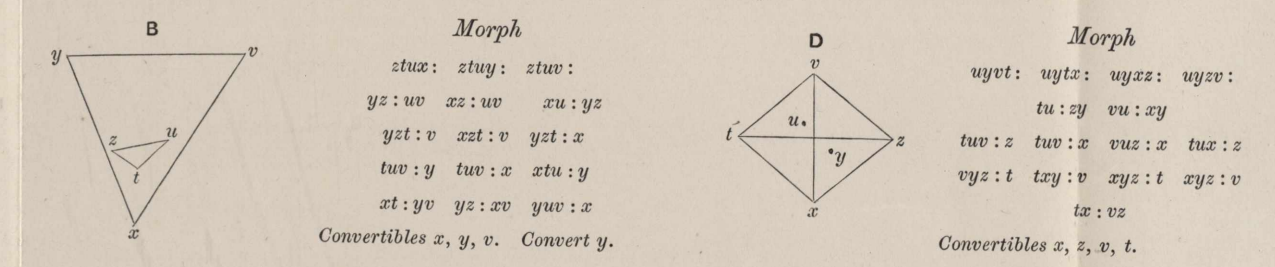
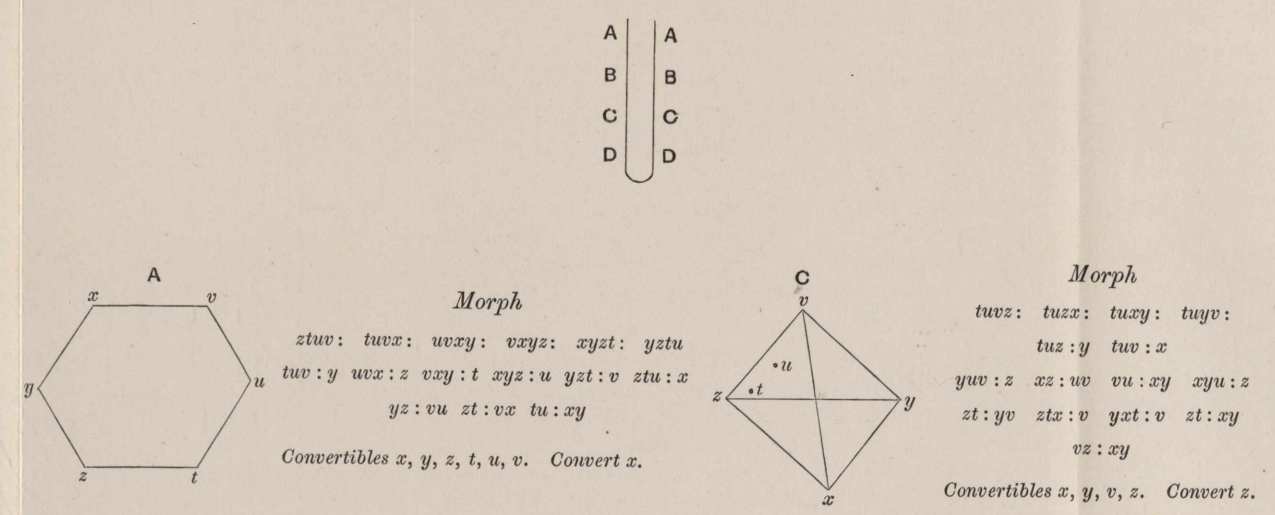
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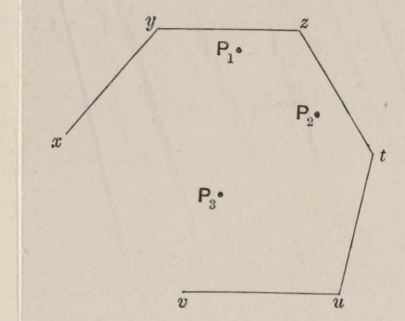
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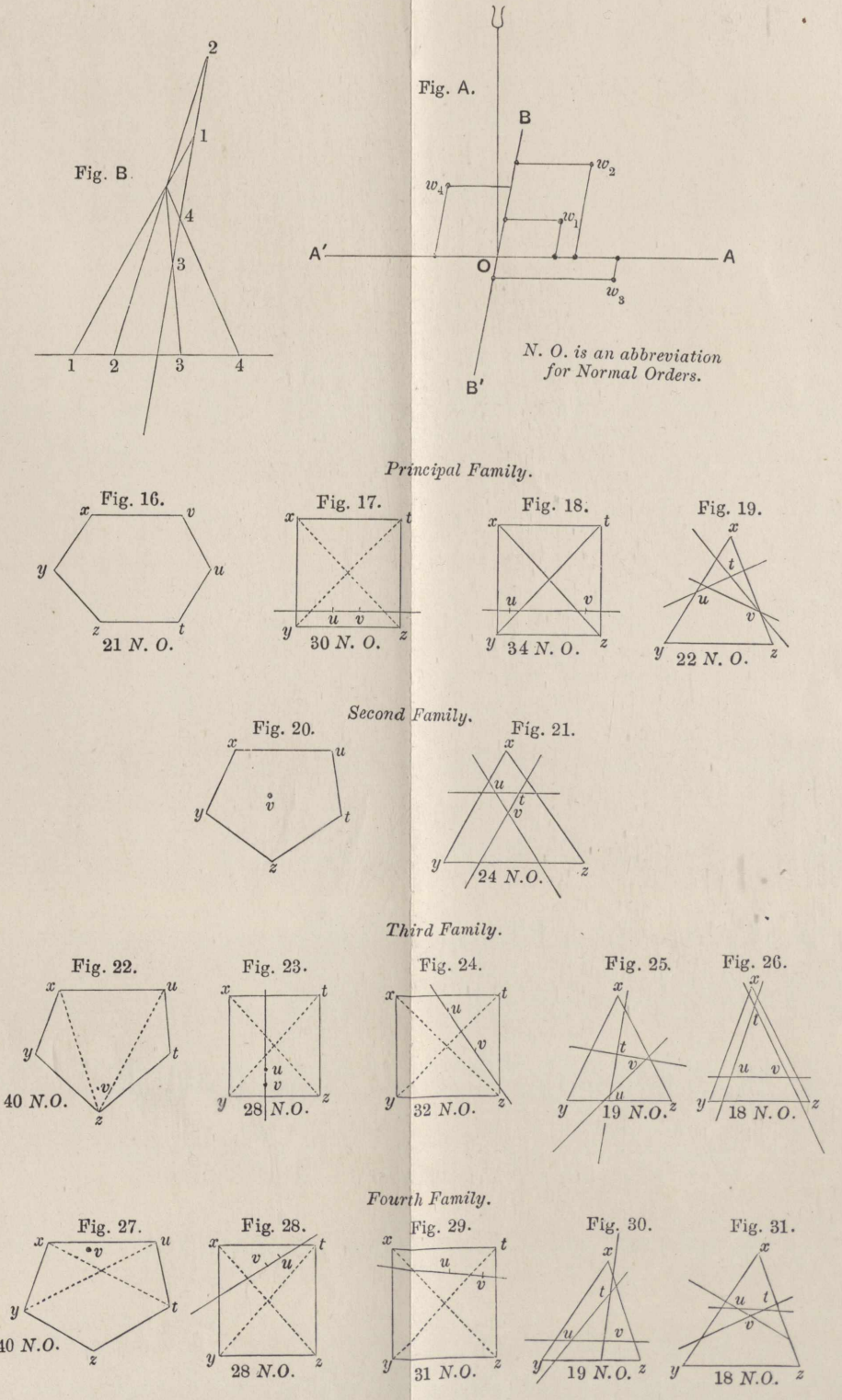
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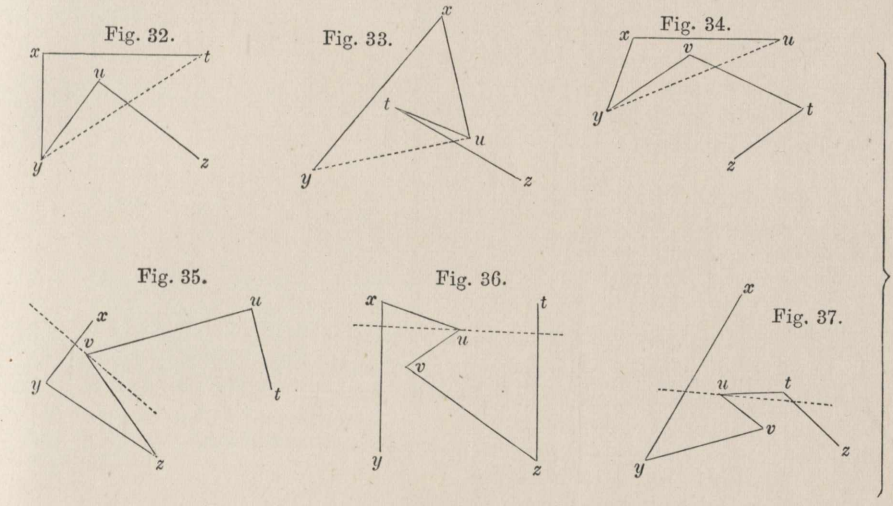
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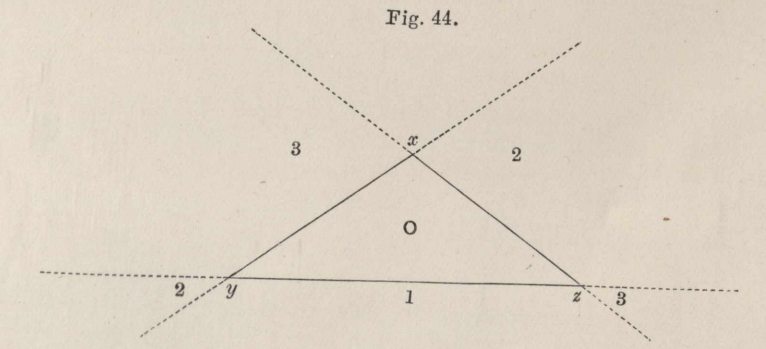
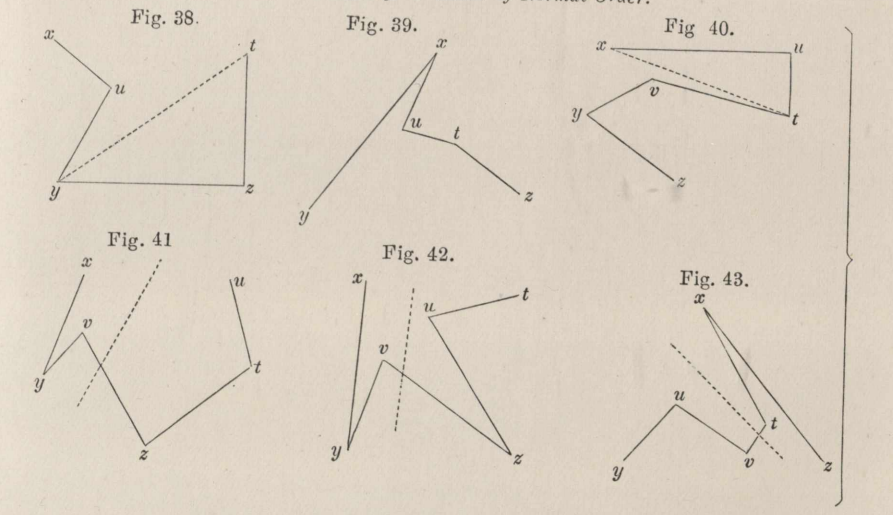
Normal Order Punctuated.
Primary at P₁ xy; ztw
P₂ xy; z; tw
P₃ xy; zt; uv



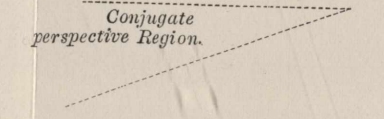
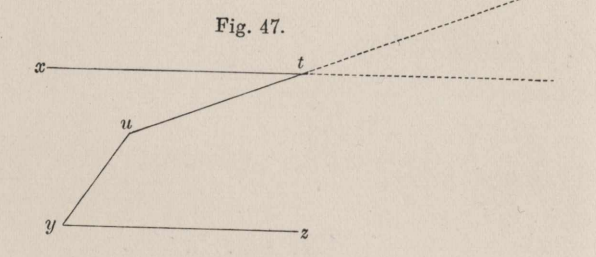
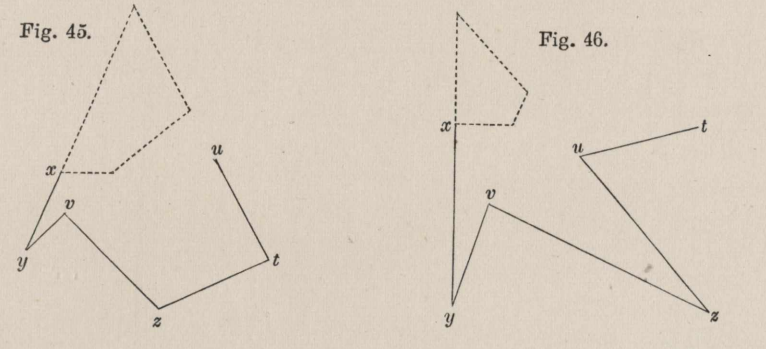
Coiled Lines or Lines of Abnormal Order



True Zig-zags or Lines of Normal Order



Perspective Regions of Normal Orders



Crossed Lines or Lines of Abnormal Order

