57.

ON THE THEORY OF ELLIPTIC FUNCTIONS.

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WE have seen [45] that the equation

$$n(n-1)x^2z+(n-1)(\alpha x-2x^3)\frac{dz}{dx}+(1-\alpha x^2+x^4)\frac{d^2z}{dx^2}-2n(\alpha^2-4)\frac{dz}{d\alpha}=0$$

is integrable, in the case of n an odd number, in the form $z=B_0+B_1x^2\ldots+B_{\frac{1}{2}(n-1)}x^{n-1}$; and the coefficients at the beginning of the series have already been determined; to find those at the end of it, the most convenient mode of writing the series will be

$$z=\mu\Sigma\,\frac{(-)^r D_r}{1.2\ldots(2r+1)}x^{n-1-2r},$$

and then the coefficients D_r are determined by

$$D_{r+2}=(2r+3)(n-2r-3)D_{r+1}\,\alpha-2n(\alpha^2-4)\frac{dD_{r+1}}{d\alpha}$$
$$-(2r+3)(2r+2)(n-2r-2)(n-2r-1)D_r.$$

The first coefficients then are

$$D_0=1,$$

$$D_1=(n-1)\alpha,$$

$$D_2=2(n-1)(n+6)+(n-1)(n-9)\alpha^2,$$

$$D_3=6(n-1)(n-9)(n+10)\alpha+(n-1)(n-9)(n-25)\alpha^3,$$

$$D_4=-36(n-1)(n^3-13n^2+36n+420)$$
$$+12(n-1)(n-9)(n-25)(n+14)\alpha^2$$
$$+(n-1)(n-9)(n-25)(n-49)\alpha^4,$$

$$D_5 = -12(n-1)(n-9)(47n^3 - 355n^2 + 3188n + 31500)\alpha$$
$$+20(n-1)(n-9)(n-25)(n-49)(n+18)\alpha^3$$
$$+(n-1)(n-9)(n-25)(n-49)(n-81)\alpha^5,$$

$$D_6 = -24(n-1)(n-9)(\ 23n^4 + 2375n^3 - 14638n^2 + 116100n + \ 693000)$$
$$-12(n-1)(n-9)(493n^4 - 8882n^3 + 70317n^2 - 361641n - 7276500)\alpha^2$$
$$+30(n-1)(n-9)(n-25)(n-49)(n-81)(n+22)\alpha^4$$
$$+(n-1)(n-9)(n-25)(n-36)(n-49)(n-81)(n-121)\alpha^6,$$
&c.

And, in general,

$$D_r = \qquad (n-1)(n-9)\ldots\{n-(2r-1)^2\}\alpha^r$$
$$+r(r-1)(n-1)(n-9)\ldots\{n-(2r-3)^2\}(n+4r-2)\alpha^{r-2},$$
&c.

(where however the next term does not contain the factor $(n-1)(n-9)\ldots\{n-(2r-5)^2\}$).

In the case when $n = \nu^2$, then in order that the constant term may reduce itself to unity, we must assume

$$\mu = (-)^{\frac{1}{2}(\nu-1)}\nu\,;$$

this is evident from what has preceded.