560.

[ADDITION TO LORD RAYLEIGH'S PAPER "ON THE NUMERICAL CALCULATION OF THE ROOTS OF FLUCTUATING FUNCTIONS."]

[From the *Proceedings of the London Mathematical Society*, vol. v. (1873—1874), pp. 123, 124. November 22.]

PROF. CAYLEY, to whom Lord Rayleigh's paper was referred, pointed out that a similar result may be attained by a method given in a paper by Encke, "Allgemeine Auflösung der numerischen Gleichungen," *Crelle*, t. XXII. (1841), pp. 193—248, as follows:

Taking the equation

$$0 = 1 - ax + bx^2 - cx^3 + dx^4 - ex^5 + fx^6 - gx^7 + hx^8 - \dots;$$

if the equation whose roots are the squares of these is

$$0 = 1 - a_1x + b_1x^2 - c_1x^3 + \dots,$$

then

$$\begin{aligned} a_1 &= a^2 - 2b, \\ b_1 &= b^2 - 2ac + 2d, \\ c_1^2 &= c^2 - 2bd + 2ae - 2f, \\ d_1^2 &= d^2 - 2ce + 2bf - 2ag + 2h, \text{ \&c.}; \end{aligned}$$

and we may in the same way derive a_2, b_2, c_2, &c. from a_1, b_1, c_1, &c., and so on.

As regards the function

$$J_n(z) = \frac{z^n}{2^n . \Gamma(n+1)} \left\{1 - \frac{z^2}{2 . 2n+2} + \frac{z^4}{2 . 4 . 2n+2 . 2n+4} - \dots\right\},$$

we have as follows:

$$a^{-1} = 2^2 . n+1,$$
$$b^{-1} = 2^5 . n+1 . n+2,$$
$$c^{-1} = 2^7 .\ 3 . n+1 \dots n+3,$$
$$d^{-1} = 2^{11} . 3 . n+1 \dots n+4,$$
$$e^{-1} = 2^{13} . 3 . 5 . n+1 \dots n+5,$$
$$f^{-1} = 2^{16} . 3^2 . 5 . n+1 \dots n+6,$$
$$g^{-1} = 2^{18} . 3^2 . 5 . 7 . n+1 \dots n+7,$$
$$h^{-1} = 2^{23} . 3^2 . 5 . 7 . n+1 \dots n+8,$$
$$a_1^{-1} = 2^4 . (n+1)^2 . n+2,$$
$$b_1^{-1} = 2^9 . (n+1 . n+2)^2 . n+3 . n+4,$$
$$c_1^{-1} = 2^{13} . 3 . (n+1 \dots n+3)^2 . n+4 \dots n+6,$$
$$d_1^{-1} = 2^{19} . 3 . (n+1 \dots n+4)^2 . n+5 \dots n+8,$$

$$a_2 = \frac{5n+11}{2^8 . (n+1)^4 (n+2)^2 n+3 . n+4},$$

$$b_2 = \frac{25n^2+231n+542}{2^{17} . (n+1 . n+2)^4 (n+3 . n+4)^2 n+5 \dots n+8},$$

$$a_3 = \frac{429n^5+7640n^4+53752n^3+185430n^2+311387n+202738}{2^{16} (n+1)^8 (n+2)^4 (n+3 . n+4)^2 n+5 . n+6 . n+7 . n+8}.$$

If $n=0$,

$$\Sigma p^{-16} = a_3 = \frac{101369}{2^{27} . 3^3 . 5 . 7} = p_1^{-16}, \text{ suppose};$$

whence

$$p_1 = 2{\cdot}404825.$$

[The quantities p_1, p_2, ... are the roots of the function $J_n(x)$ in increasing order of magnitude, so that, as these roots are all real, it follows that for $J_0(x)$,

$$a = \Sigma p_1^{-2}, \quad a_1 = \Sigma p_1^{-4}, \quad a_2 = \Sigma p_1^{-8}, \quad a_3 = \Sigma p_1^{-16}, \dots]$$