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NOTE ON THE TRANSFORMATION OF TWO SIMULTANEOUS EQUATIONS.

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WRITING in Mr Walton's equations (1) and (2)

$$\frac{a}{d},\ \frac{b}{d},\ \frac{c}{d},\ \frac{\alpha}{\delta},\ \frac{\beta}{\delta},\ \frac{\gamma}{\delta}$$

instead of a, b, c, α, β, γ respectively; and putting for shortness

$$\begin{aligned} A &= b\gamma - c\beta, & F &= a\delta - d\alpha, \\ B &= c\alpha - a\gamma, & G &= b\delta - d\beta, \\ C &= a\beta - b\alpha, & H &= c\delta - d\gamma, \end{aligned}$$

the equations become

$$\frac{a(b-c)}{F} + \frac{b(c-a)}{G} + \frac{c(a-b)}{H} = 0,$$

$$\frac{\alpha(\beta-\gamma)}{F} + \frac{\beta(\gamma-\alpha)}{G} + \frac{\gamma(\alpha-\beta)}{H} = 0.$$

Multiplying by FGH and effecting some obvious transformations, the equations become

$$\left.\begin{aligned} aAF + bBG + cCH &= 0 \\ \alpha AF + \beta BG + \gamma CH &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(18);$$

whence also

$$AF^2 + BG^2 + CH^2 = 0 \quad \ldots\ldots\ldots\ldots(19).$$

Now regarding $(\alpha, \beta, \gamma, \delta)$ as the coordinates of a point in space, the equations (18) and (19) represent each of them a cone having for vertex the point $\alpha : \beta : \gamma : \delta = a : b : c : d$, viz. (18) is a quadric cone, (19) a cubic cone; they intersect therefore in six lines; and it may be shown that these are

the line	$\alpha : \beta : \gamma = a : b : c$	(twice)	2
"	$\beta : \gamma : \delta = b : c : d$		1
"	$\gamma : \alpha : \delta = c : a : d$		1
"	$\alpha : \beta : \delta = a : b : d$		1
"	$\beta-\gamma : \gamma-\alpha : \alpha-\beta : \delta = b-c : c-a : a-b : d$		1
			6,

agreeing with Mr Walton's result.

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