583.

ON SPHEROIDAL TRIGONOMETRY.

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THE fundamental formulæ of Spheroidal Trigonometry are those which belong to a right-angled triangle PSS_0, where P is the pole, PS, PS_0 arcs of meridian, and SS_0 a geodesic line cutting the meridian PS at a given angle, and the meridian PS_0 at right angles. We consider a spherical triangle PSS_0,

$$\begin{array}{llllll} \text{Sides} & PS, & PS_0, & SS_0 = \gamma, & \gamma_0, & s, \\ \text{Angles} & S_0, & S, & P = 90^\circ, & \theta, & l, \end{array}$$

where γ is the reduced colatitude of the point S on the spheroid (and thence also γ_0 the reduced colatitude of S_0) and θ the azimuth of the geodesic SS_0, or angle at which this cuts the meridian SP; and then if S be the length of the geodesic SS_0 measured as a circular arc, radius = Earth's equatoreal radius, and L be the angle SPS_0, S, L differ from the corresponding spherical quantities s, l by terms involving the excentricity of the spheroid, viz. calling this e and writing

$$k = \frac{e \cos \gamma_0}{\sqrt{1 - e^2 \sin^2 \gamma_0}},$$

then (see Hansen's "Geodätische Untersuchungen," *Abh. der K. Sächs. Gesell.*, t. VIII. (1865) pp. 15 and 23, but using the foregoing notation) we have, to terms of the sixth order in e,

$$\begin{aligned} \frac{S}{\sqrt{1-e^2}} = \; & (1 + \tfrac{1}{4}k^2 + \tfrac{13}{64}k^4 + \tfrac{45}{256}k^6)\, s \\ & + (\tfrac{1}{8}k^2 + \tfrac{3}{32}k^4 + \tfrac{79}{1024}k^6) \sin 2s \\ & + (\tfrac{1}{256}k^4 + \tfrac{5}{1024}k^6) \sin 4s \\ & + \tfrac{1}{3072}k^6 \sin 6s\,; \end{aligned}$$

and

$$\begin{aligned} L = l - \tfrac{1}{2}e^2 \sin \gamma_0 \{ & (1 - \tfrac{1}{8}k^2 + \tfrac{1}{4}e^2 - \tfrac{5}{64}k^4 + \tfrac{1}{8}e^4)\, s \\ & - (\tfrac{1}{16}k^2 + \tfrac{1}{32}k^4) \sin 2s \\ & + \tfrac{1}{256}k^4 \sin 4s \}, \end{aligned}$$

which are the formulæ in question.