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THEOREM ON THE *n*th ROOTS OF UNITY.

[From the *Messenger of Mathematics*, vol. IV. (1875), p. 171.]

IF n be an odd prime, and α an imaginary nth root of unity, then

$$(-)^{\frac{1}{2}(n-1)}\, n - 1 = 4\left\{\frac{\alpha}{1+\alpha^2} + \frac{\alpha^2}{1+\alpha^4} + \frac{\alpha^3}{1+\alpha^6} \ldots + \frac{\alpha^{\frac{1}{2}(n-1)}}{1+\alpha^{n-1}}\right\};$$

for instance,

$$n = 3, \quad -4 = 4\,\frac{\alpha}{1+\alpha^2},$$

verified at once by means of the equation $1 + \alpha + \alpha^2 = 0$:

$$n = 5, \quad 4 = 4\left(\frac{\alpha}{1+\alpha^2} + \frac{\alpha^2}{1+\alpha^4}\right),$$

where the term in ( ) is

$$\frac{\alpha(1+\alpha^4) + \alpha^2(1+\alpha^2)}{(1+\alpha^2)(1+\alpha^4)},$$

that is,

$$= \frac{\alpha + 1 + \alpha^2 + \alpha^4}{1 + \alpha^2 + \alpha^4 + \alpha}, \quad = 1:$$

and so in other cases.