609.

ON THE ANALYTICAL FORMS CALLED FACTIONS.

[From the *Report of the British Association for the Advancement of Science*, (1875), p. 10.]

A FACTION is a product of differences such that each letter occurs the same number of times; thus we have a quadrifaction where each letter occurs twice, a cubifaction where each letter occurs three times, and so on. A broken faction is one which is a product of factions having no common letter; thus

$$(a-b)^2(c-d)(d-e)(e-c)$$

is a broken quadrifaction, the product of the quadrifactions

$$(a-b)^2 \text{ and } (c-d)(d-e)(e-c).$$

We have, in regard to quadrifactions, the theorem that every quadrifaction is a sum of broken quadrifactions such that each component quadrifaction contains two or else three letters. Thus we have the identity

$$2(a-b)(b-c)(c-d)(d-a)=(b-c)^2.(a-d)^2-(c-a)^2.(b-d)^2+(a-b)^2.(c-d)^2,$$

which verifies the theorem in the case of a quadrifaction of four letters; but the verification even in the next following case of a quadrifaction of five letters is a matter of some difficulty.

The theory is connected with that of the invariants of a system of binary quantics.