

142.

NUMERICAL TABLES SUPPLEMENTARY TO SECOND MEMOIR
ON QUANTICS.

[Now first published (1889).]

In the present paper I arrange in a more compendious form and continue to a much greater extent the tables (first of each pair) given Nos. 35—39 of my Second Memoir on Quantics, 141, pp. 260—264, which relate to the cubic, the quartic and the quintic functions; and I give the like tables for the sextic, the septimic and the octavic functions respectively. The cubic table exhibits the coefficients of the several xz terms of the function $1 \div (1 - z.1 - xz.1 - x^2z.1 - x^3z)$, or, what is the same thing, it gives the number of partitions of a given number into a given number of parts, the parts being 0, 1, 2, 3, (repetitions admissible): or again, regarding the letters a, b, c, d , as having the weights 0, 1, 2, 3 respectively, it shows the number of literal terms of a given degree and given weight. And similarly for the quartic, quintic, sextic, septimic and octavic tables respectively, the parts of course being 0, 1, ... up to 4, 5, 6, 7 or 8, and the letters being a, b, \dots up to e, f, g, h or i . The extent of the tables is as follows:

cubic table extends to deg-weight	18—27
quartic " " " "	18—36
quintic " " " "	18—45
sextic " " " "	15—45
septimic " " " "	12—42
octavic " " " "	10—40

viz. for the quintic, the sextic and the octavic functions these are the deg-weights of the highest invariants respectively. I designate the Tables as the ad -, ae -, af -, ag -, ah - and ai -tables respectively.

It is to be noticed that in the several tables the lower part of each column is for shortness omitted; the column has to be completed by taking into it the series

of bottom terms of each of the preceding columns: thus in the *af*- or quintic table the complete column for degree 3 would be

D	3
W	8-7
-0	6
-1	6
-2	5
-3	4
-4	3
-5	2
-6	1
-7	1

where the concluding terms 2, 1, 1 are the bottom terms of the three preceding columns respectively. And the meaning is that for degree 3, and weight 8, or 7, the number of terms is =6; for weight 7-1, =6, the number of terms is =6; and similarly for weights 5, 4, 3, 2, 1, 0 the numbers are 5, 4, 3, 2, 1, 1; the numbers are those of the terms

W.	0	1	2	3	4	5	6	7	8
	a^3	a^2b	a^2c ab^2	a^2d abc b^3	a^2e abd ac^2 b^2c	a^2f abe acd b^2d bc^2	abf ace ad^2 b^2e bcd c^3	acf ade b^2f b^2ce bd^2 c^2d	adf ae^2 bcf bde c^2e cd^2
No.	1	1	2	3	4	5	6	6	6

The like remarks and explanations apply to the other tables.

ad-TABLE.

D	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
W	0	2-1	3	5-4	6	8-7	9	11-10	12	14-13	15	17-16	18	20-19	21	23-22	24	26-25	27	
	1	1	2	3	5	6	8	10	13	15	18	21	25	28	32	36	41	45	50	-0
			2	3	4	6	8	10	12	15	18	21	24	28	32	36	40	45	50	-1
				4	5	7	9	12	14	17	20	24	27	31	35	40	44	49	49	-2
					7	8	11	13	17	19	23	26	31	34	39	43	49	49	49	-3
						10	12	15	18	22	25	29	33	38	42	47	47	47	47	-4
							14	16	20	23	28	31	36	40	46	46	46	46	46	-5
								19	21	26	29	35	38	44	44	44	44	44	44	-6
									24	27	32	36	42	42	42	42	42	42	42	-7
										30	33	39	39	39	39	39	39	39	39	-8
											37	37	37	37	37	37	37	37	37	-9

ae-TABLE.

D	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
W	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	
	1	1	3	5	8	12	18	24	33	43	55	69	86	104	126	150	177	207	241	-0
		1	2	4	7	11	16	23	31	41	53	67	83	102	123	147	174	204	237	-1
			2	4	7	11	16	23	31	41	53	67	83	102	123	147	174	204	237	-2
				3	5	9	14	20	28	38	49	63	79	97	118	142	168	198	231	-3
					5	8	13	19	27	36	48	61	77	95	116	139	166	195	228	-4
						6	10	16	23	32	43	56	71	89	109	132	158	187	219	-5
							9	14	21	30	40	53	68	85	105	128	153	182	214	-6
								11	17	25	35	47	61	78	97	119	144	172	203	-7
									15	22	32	43	57	73	92	113	138	165	196	-8
										18	26	37	50	65	83	104	127	154	184	-9
											23	33	45	60	77	97	120	146	175	-10
												27	38	52	68	87	109	134	162	-11
													34	46	62	80	101	125	153	-12
														39	53	70	90	113	139	-13
															47	63	82	104	129	-14
																54	71	92	116	-15
																	64	83	106	-16
																		72	93	-17
																			84	-18

af-TABLE.

D	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18										
W	0	3-2	5	8-7	10	13-12	15	18-17	20	23-22	25	28-27	30	33-32	35	38-37	40	43-42	45										
	1	1	3	6	12	20	32	49	73	102	141	190	252	325	414	521	649	795	967	-0									
		1	3	6	11	19	32	48	71	101	141	188	249	322	414	518	645	791	966	-1									
			2	5	11	18	30	46	70	98	137	184	247	317	408	511	641	783	957	-2									
				2	4	9	16	29	43	66	93	134	178	240	309	402	501	630	770	948	-3								
					3	8	14	25	39	63	88	127	170	233	299	390	488	619	754	930	-4								
						6	11	23	35	57	81	121	161	222	286	379	472	601	734	912	-5								
							5	9	19	30	52	74	111	150	212	272	362	453	583	711	886	-6							
								7	16	26	45	66	103	139	197	256	346	433	559	684	860	-7							
									12	21	40	58	92	126	184	239	325	409	536	655	827	-8							
										10	17	33	50	83	114	168	220	306	385	507	623	795	-9						
											13	28	43	72	101	154	202	283	359	480	590	756	-10						
												22	35	63	89	137	183	262	333	448	554	719	-11						
													18	29	53	77	123	165	238	306	419	519	677	-12					
														23	45	66	107	146	217	280	386	482	637	-13					
															36	55	94	129	194	253	356	446	593	-14					
																30	46	80	112	174	228	324	409	553	-15				
																	37	68	97	152	203	295	374	509	-16				
																		56	82	134	180	264	339	469	-17				
																			47	69	115	157	237	306	427	-18			
																				57	99	137	209	273	389	-19			
																					83	117	185	243	350	-20			
																						70	100	160	214	315	-21		
																							84	139	188	279	-22		
																									118	162	248	-23	
																										101	140	217	-24
																											119	190	-25
																												163	-26
																												141	-27

ag-TABLE.

D	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
W	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	
1	1	4	8	18	32	58	94	151	227	338	480	676	920	1242	1636	-0	
	1	3	8	16	32	55	94	147	227	332	480	668	920	1232	1635	-1	
	1	3	7	16	30	55	90	146	221	330	471	664	907	1226	1617	-2	
		2	7	14	29	51	88	139	217	319	464	648	896	1203	1601	-3	
		2	5	13	25	48	81	134	205	310	446	634	870	1182	1565	-4	
			4	10	23	42	76	123	196	293	431	608	847	1145	1533	-5	
			3	9	19	39	68	116	182	280	408	587	813	1113	1483	-6	
				6	16	32	61	103	169	258	387	553	780	1064	1435	-7	
				5	12	28	52	94	152	241	359	525	737	1021	1373	-8	
					10	22	46	81	139	218	335	488	699	965	1316	-9	
					7	18	37	71	121	199	304	455	650	914	1244	-10	
						13	31	59	107	175	278	415	607	852	1178	-11	
						11	24	51	91	157	248	382	557	798	1102	-12	
							19	40	78	134	222	341	512	733	1031	-13	
							14	33	64	117	193	308	462	677	952	-14	
								25	54	98	170	271	419	614	882	-15	
								20	42	83	144	240	371	559	803	-16	
									34	67	124	206	331	499	734	-17	
									26	56	103	180	289	449	661	-18	
										43	86	151	253	394	596	-19	
										35	69	129	216	349	529	-20	
											57	106	187	302	472	-21	
											44	88	156	263	412	-22	
												70	132	223	362	-23	
												58	108	192	303	-24	
													89	159	270	-25	
													71	134	228	-26	
														109	195	-27	
														90	161	-28	
															135	-29	
															110	-30	

ah-TABLE.

D	0	1	2	3	4	5	6	7	8	9	10	11	12	
W	0	4-3	7	11-10	14	18-17	21	25-24	28	32-31	35	39-38	42	
	1	1	4	10	24	49	94	169	289	468	734	1117	1656	-0
		1	4	10	23	48	94	166	285	464	734	1109	1646	-1
		1	3	9	23	46	90	162	282	454	722	1093	1634	-2
			3	8	20	43	88	155	272	441	709	1069	1605	-3
			2	7	19	39	81	146	263	424	686	1038	1572	-4
			2	5	16	35	76	136	247	403	663	1000	1524	-5
				4	14	30	68	125	233	379	629	957	1475	-6
				3	11	26	61	112	214	354	598	908	1410	-7
					9	21	52	100	197	325	558	856	1346	-8
					6	17	46	87	176	297	520	799	1271	-9
					5	13	37	75	158	268	477	742	1197	-10
						10	31	63	137	239	437	682	1114	-11
						7	24	53	120	210	392	623	1036	-12
							19	42	101	184	353	563	950	-13
							14	34	86	157	311	506	871	-14
							11	26	70	134	274	449	788	-15
								20	58	112	236	397	711	-16
								15	45	93	204	346	633	-17
									36	75	171	300	564	-18
									27	61	145	256	493	-19
									21	47	119	218	432	-20
										37	98	182	372	-21
										28	78	152	320	-22
											63	124	270	-23
											48	101	229	-24
											38	80	189	-25
												64	157	-26
												49	127	-27
													103	-28
													81	-29
													65	-30

ai-TABLE.

	0	1	2	3	4	5	6	7	8	9	10
W	0	4	8	12	16	20	24	28	32	36	40
1	1	5	13	33	73	151	289	526	910	1514	-0
	1	4	12	31	71	147	285	519	902	1502	-1
	1	4	12	31	70	146	282	515	894	1492	-2
	1	3	11	28	66	139	272	499	873	1460	-3
		3	10	27	63	134	263	486	851	1430	-4
		2	8	23	57	123	247	461	816	1379	-5
		2	7	21	52	116	233	440	783	1331	-6
			5	17	45	103	214	409	738	1265	-7
			4	15	40	94	197	383	696	1214	-8
			3	11	33	81	176	348	645	1127	-9
				9	28	71	158	319	597	1057	-10
				6	22	59	137	284	543	974	-11
				5	18	51	120	255	495	900	-12
					13	40	101	221	441	816	-13
					10	33	86	194	394	742	-14
					7	25	70	164	345	662	-15
						20	58	141	302	593	-16
						14	45	116	258	519	-17
						11	36	97	222	457	-18
							27	77	185	393	-19
							21	63	156	340	-20
							15	48	127	286	-21
								38	104	243	-22
								28	82	200	-23
								22	66	167	-24
									50	134	-25
									39	109	-26
									29	85	-27
										68	-28
										51	-29
										40	-30

The numbers of each table are connected in several ways with those of the preceding tables. One of these connexions, which is of some importance, is best explained by an example: in the *af*-table, 8-20, the number of terms of degree 8 and weight 20 is 73; and we have $73 = 1 + 6 + 16 + 23 + 27$, viz. (see p. 288) these are the numbers of the terms in a^4, a^3, a^2, a^1, a^0 respectively: the complementary factors, (for example) of a^3 are bef^3 , &c. terms in b, c, d, e, f of the degree 5 and weight 20, and (replacing therein each letter by that which immediately precedes it) these are in number equal to the terms in a, b, c, d, e of the degree 5 and weight $20 - 5 = 15$; thus the number 6 of the terms in question is that for the deg-weight 5-15 of the *ae*-table: and so 1, 6, 16, 23, 27 are the numbers in the *ae*-table for the deg-weights 4-16, 5-15, 6-14, 7-13 and 8-12 respectively, or (making a change rendered necessary by the abbreviated form of the tables) say for the deg-weights 4-0, 5-10, 6-14, 7-13 and 8-12.