151.

TABLES OF THE STURMIAN FUNCTIONS FOR EQUATIONS OF THE SECOND, THIRD, FOURTH, AND FIFTH DEGREES.

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THE general expressions for the Sturmian functions in the form of determinants are at once deducible from the researches of Professor Sylvester in his early papers on the subject in the *Philosophical Magazine*, and in giving these expressions in the Memoir 'Nouvelles Recherches sur les Fonctions de M. Sturm,' *Liouville*, t. XIII. p. 269 (1848), [65], I was wrong in claiming for them any novelty. The expressions in the last-mentioned memoir admit of a modification by which their form is rendered somewhat more elegant; I propose on the present occasion merely to give this modified form of the general expression, and to give the developed expressions of the functions in question for equations of the degrees two, three, four, and five.

Consider in general the equation

$$U = (a,\ b,\ \dots\ j,\ k \between x,\ 1)^n,$$

and write

$$P = (a,\ b,\ \dots\ \ \ j \between x,\ 1)^{n-1},$$

$$Q = (b,\ \ \ \dots\ j,\ k \between x,\ 1)^{n-1},$$

then supposing as usual that the first coefficient a is positive, and taking for shortness n_1, n_2, &c. to represent the binomial coefficients $\frac{n-1}{1}$, $\frac{n-1 \, . \, n-2}{1 \, . \, 2}$, &c. corresponding to the index $(n-1)$, the Sturmian functions, each with its proper sign, are as follows, viz.

$$U,\ P,\ \begin{vmatrix} P, & Q \\ a, & b \end{vmatrix},\ -\begin{vmatrix} xP, & P, & xQ, & Q \\ a, & ., & b, & . \\ n_1b, & a, & n_1c, & b \\ n_2c, & n_1b, & n_2d, & n_1c \end{vmatrix},$$

$$+\begin{vmatrix} x^2P, & xP, & P, & x^2Q, & xQ, & Q \\ a, & ., & ., & b, & ., & . \\ n_1b, & a, & ., & n_1c, & b, & . \\ n_2c, & n_1b, & a, & n_2d, & n_1c, & b \\ n_3d, & n_2c, & n_1b, & n_3e, & n_2d, & n_1c \\ n_4e, & n_3d, & n_2c, & n_4f, & n_3e, & n_2d \end{vmatrix},\ \&c.$$

where the terms containing the powers of x, which exceed the degrees of the several functions respectively, vanish identically (as is in fact obvious from the form of the expressions), but these terms may of course be omitted *ab initio*.

The following are the results which I have obtained; it is well known that the last or constant function is in each case equal to the discriminant, and as the expressions for the discriminant of equations of the fourth and fifth degrees are given, Tables No. 12 and No. 26 [Q', see 143] in my 'Second Memoir upon Quantics'(¹), I have thought it sufficient to refer to these values without repeating them at length.

Table for the degree 2.

The Sturmian functions for the quadric $(a, b, c \between x, 1)^2$ are

$$\left(\begin{array}{|c|c|c|}\hline a+1 & b+2 & c+1 \\ \hline\end{array} \between x,\ 1\right)^2,$$

$$\left(\begin{array}{|c|c|}\hline a+1 & b+1 \\ \hline\end{array} \between x,\ 1\right),$$

$$\begin{array}{|c|}\hline ac-1 \\ b^2+1 \\ \hline\end{array}$$

Table for the degree 3.

The Sturmian functions for the cubic $(a, b, c, d \between x, 1)^3$ are

$$\left(\begin{array}{|c|c|c|c|}\hline a+1 & b+3 & c+3 & d+1 \\ \hline\end{array} \between x,\ 1\right)^3,$$

¹ *Philosophical Transactions*, t. CXLVI. p. 101 (1856), [141].

$$\left(\begin{array}{|c|c|c|} \hline a+1 & b+2 & c+1 \\ \hline \end{array}\big)\!\big(x,\ 1)^2,$$

$$\left(\begin{array}{|c|c|} \hline ac-2 & ad-1 \\ b^2+2 & bc+1 \\ \hline \end{array}\big)\!\big(x,\ 1),$$

$$\begin{array}{|l|} \hline a^2d^2\ +1 \\ abcd+6 \\ ac^3\ \ -4 \\ bd^3\ \ -4 \\ b^2c^2\ -3 \\ \hline \end{array}$$

Table for the degree 4.

The Sturmian functions for the quartic $(a,\ b,\ c,\ d,\ e\big)\!\big(x,\ 1)_4$ are

$$\left(\begin{array}{|c|c|c|c|c|} \hline a+1 & b+4 & c+6 & d+4 & e+1 \\ \hline \end{array}\big)\!\big(x,\ 1)^4,$$

$$\left(\begin{array}{|c|c|c|c|} \hline a+1 & b+3 & c+3 & d+1 \\ \hline \end{array}\big)\!\big(x,\ 1)^3,$$

$$\left(\begin{array}{|c|c|c|} \hline ac-3 & ad-3 & ae-1 \\ b^2\ +3 & bc\ +3 & bd+1 \\ \hline \end{array}\big)\!\big(x,\ 1)^2,$$

$$3\left(\begin{array}{|l|l|} \hline a^2ce\ \ -\ \ 1 & a^2de\ +1 \\ a^2d^2\ \ +\ \ 3 & abce\ -4 \\ ab^2e\ \ +\ \ 1 & abd^2\ -1 \\ abcd-14 & ac^2d\ +3 \\ ac^3\ \ \ +\ \ 9 & b^3e\ \ \ +3 \\ b^3d\ \ \ +\ \ 8 & b^2cd\ -2 \\ b^2c^2\ \ -\ \ 6 & \\ \hline \end{array}\big)\!\big(x,\ 1),$$

$$-\begin{array}{|l|} \hline a^3e^3+1 \\ \&c. \\ \text{Disct. Tab.} \\ \text{No. 12.} \\ \hline \end{array}\ .$$

Table for the degree 5.

The Sturmian functions for the quintic $(a,\ b,\ c,\ d,\ e,\ f\big)\!\big(x,\ 1)^5$ are

$$\left(\begin{array}{|c|c|c|c|c|c|} \hline a+1 & b+5 & c+10 & d+10 & e+5 & f+1 \\ \hline \end{array}\big)\!\big(x,\ 1)^5,$$

(

$a+1$	$b+4$	$c+6$	$d+4$	$e+1$

$\between x, 1)^4$,

(

$ac-4$ b^2+4	$ad-6$ $bc+6$	$ae-4$ $bd+4$	$af-1$ $be+1$

$\between x, 1)^3$,

2 (

a^2ce-8	a^2cf-2	a^2df+3
a^2d^2+18	a^2de+12	$abcf-11$
ab^2e+8	ab^2f+2	$abde-3$
$abcd-76$	$abce-42$	ac^2e+8
ac^3+48	abd^2-12	b^3f+8
b^3d+40	ac^2d+32	b^2ce-5
b^2c^2-30	b^3e+30	
	b^2cd-20	

$\between x, 1)^2$,

2 (

a^3cf^2	−	2	a^3df^2	+	3
a^3def	+	24	a^3e^2f	−	8
a^3e^3	−	32	a^2bcf^2	−	11
$a^2b^2f^2$	+	2	a^2bdef	+	58
a^2bde^2	+	264	a^2be^3	+	8
a^2bcef	−	52	a^2c^2ef	+	104
a^2bd^2f	−	96	a^2cd^2f	−	156
a^2c^2df	+	64	a^2cde^2	−	96
$a^2c^2e^2$	+	352	a^2d^3e	+	108
a^2cd^2e	−	938	ab^3f^2	+	8
a^2d^4	+	432	ab^2cef	−	266
ab^3ef	+	28	ab^2d^2f	−	8
ab^2ce^2	−	970	ab^2de^2	+	35
ab^2d^2e	+	120	abc^2df	+	584
abc^2de	+	2480	abc^2e^2	+	120
ab^2cdf	+	264	$abcd^2e$	−	360
$abcd^3$	−	1440	ac^4f	−	288
abc^3f	−	192	ac^3de	+	160
ac^4e	−	960	b^4ef	+	120
ac^3d^2	+	640	b^3cdf	−	320
b^4df	−	160	b^3ce^2	−	75
b^4e^2	+	450	b^3d^2e	+	200
b^3cde	−	1400	b^2c^3f	+	180
b^3d^3	+	800	b^2c^2de	−	100
b^3c^2f	+	120			
b^2c^3e	+	600			
$b^2c^2d^2$	−	400			

$\between x, 1)$,

a^4f^4+1 + &c. Disct. Tab. No. 26, [Q′].