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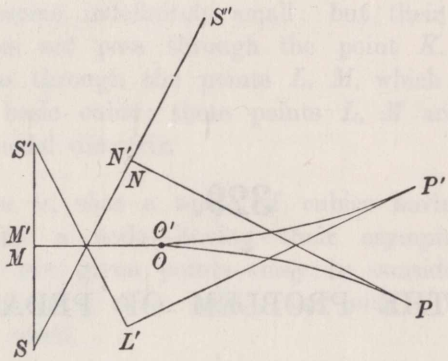
NOTE ON THE PROBLEM OF PEDAL CURVES.

[From the *Philosophical Magazine*, vol. XXVI. (1863), pp. 20, 21.]

It is not, so far as I am aware, generally known that the problem of pedal curves (Steiner's *Fusspuncten-Curve*) was considered by Maclaurin in the *Geometria Organica*, 1720. He appears to have been led to it through an idea such as Sir W. R. Hamilton's *Hodograph*, or at any rate with a view to a dynamical application, for he remarks, p. 95, "Cum vero geometria quæ curvas ad datum centrum relatas contemplatur in philosophia naturali ad motus corporum et vires evolvendas facilius applicari possit,...hac sectione considerabimus curvas tanquam ad punctum quodvis datum relatas ex quo ad omnia circumferentiæ puncta radii undique educuntur, et simul perpendicula in illorum punctorum tangentes demittuntur, et rationem radii ad perpendiculum tanquam curvæ characterem usurpabimus." And accordingly, Props. IX. to XII., he considers the problem: Given a point S in the plane of a given curve, to find the locus of the intersection of a tangent of the curve by the perpendicular let fall upon it from the point S; with some special cases, and deductions from it. In particular if the given curve be a circle, the locus in question (or pedal curve) is a curve of the fourth order having a double point S; viz. if S be inside the circle, this is a conjugate or isolated point; but if outside, a double point with two real branches: if S be on the circle, then instead of the double point we have a cusp: it is shown that in each case the pedal curve is in fact an epicycloid. If the given curve be a parabola, then the locus or pedal curve is a curve of the third order, viz. a defective hyperbola having a double point at S, and with its single asymptote perpendicular to the axis of the parabola: some particular cases are specially noticed. If the curve be an ellipse or hyperbola, then, as in the case of the circle, the locus or pedal curve is a curve of the fourth order having a double point at S. And it is moreover shown, Prop. XII., that for any given curve whatever the locus or pedal curve is, in a generalized sense of the term, an epicycloid. This is in fact seen very easily by a mere inspection of the figure. Imagine the curve $O'P'$, rigidly connected with and carrying along with it the point S', to roll on the similar and equal fixed curve OP symmetrically situate on the other side of the common tangent OM or OM'; then when P' coincides with P, the point S' is brought to S'', where $SNN'S''$ is the perpendicular from S on the tangent PN or PN', and $SN = N'S''$, that is, $SS'' = 2SN$;



and the curve generated by S'' (that is S'), or say the epicycloid the locus of S', is a curve similar to and similarly situate with the pedal curve the locus of N, but of twice the linear magnitude of the pedal curve. Or, what is the same thing, if instead of the given curve we consider a similar and similarly situated curve of twice the linear magnitude (the point S being the centre of similitude), then the epicycloid the locus of S' is the pedal curve of the substituted curve in relation to the point S. It may be added that, in accordance with a theorem of Dandelin's, if rays proceeding from the point S are reflected at the given curve, then the epicycloid (or pedal) in question is the *secondary caustic*, or an orthogonal trajectory of the reflected rays.

2, *Stone Buildings, W.C., June* 3, 1863.