240.

NOTE ON A THEOREM IN SPHERICAL TRIGONOMETRY.

[From the *Philosophical Magazine*, vol. XVII. (1859), p. 151.]

I AM not aware that the following theorem has been noticed: viz., in any spherical triangle, if as usual a, b, c are the sides, and A, B, C the opposite angles, then

$$\sin b \sin c + \cos b \cos c \cos A = \sin B \sin C - \cos B \cos C \cos a,$$
$$\sin c \sin a + \cos c \cos a \cos B = \sin C \sin A - \cos C \cos A \cos b,$$
$$\sin a \sin b + \cos a \cos b \cos C = \sin A \sin B - \cos A \cos B \cos c.$$

The demonstration is very simple; in fact we have

$$\begin{aligned}
&\sin b \sin c + \cos b \cos c \cos A \\
&= \sin b \sin c (\sin^2 A + \cos^2 A) + \cos b \cos c \cos A \\
&= \sin b \sin c \sin^2 A + \cos A (\cos b \cos c + \sin b \sin c \cos A) \\
&= \sin B \sin C \sin^2 a + \cos A \cos a \\
&= \sin B \sin C (1 - \cos^2 a) + \cos A \cos a \\
&= \sin B \sin C + \cos a (\cos A - \sin B \sin C \cos a) \\
&= \sin B \sin C - \cos B \cos C \cos a,
\end{aligned}$$

which proves the theorem.

2, *Stone Buildings, W.C., January* 5, 1859.

A geometrical proof and interpretation are given, G. B. Airy, "Remarks on Mr Cayley's Trigonometrical Theorem, etc." *Phil. Mag.* same volume, p. 176. I transfer to this place the concluding sentence of the subsequent paper 243. "I take the opportunity of noticing that the theorem in spherical trigonometry, which I gave in the February Number, is not new, but, as pointed out by Prof. Chauvenet in the Mathematical Monthly (Cambridge, U.S.), is to be found in Cagnoli's 'Trigonometry' (1808)."