803.

ON MR ANGLIN'S FORMULA FOR THE SUCCESSIVE POWERS OF THE ROOT OF AN ALGEBRAICAL EQUATION.

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SUPPOSE $x^m - px^{m-1} + qx^{m-2} - \ldots = 0$, then the successive powers x^m, x^{m+1}, x^{m+2}, &c. of x can be expressed in the form $Px^{m-1} - Qx^{m-2} + Rx^{m-3} - \&c.$ Mr Anglin has obtained for this purpose a very elegant formula, with a demonstration which (it occurred to me) might be presented under a somewhat simplified form; and he has permitted me to draw up the present Note.

Take, for greater convenience, the equation to be

$$x^4 - px^3 + qx^2 - rx + s = 0,$$

and let $h_1, h_2, h_3, \ldots$ be the sums of the homogeneous products of the roots, of the orders 1, 2, 3, &c. respectively; then, writing also $h_0 = 1$, we have

$$\begin{aligned} h_1 &= h_0 p, \\ h_2 &= h_1 p - h_0 q, \\ h_3 &= h_2 p - h_1 q + h_0 r, \\ h_4 &= h_3 p - h_2 q + h_1 r - h_0 s, \\ h_5 &= h_4 p - h_3 q + h_2 r - h_1 s, \\ &\vdots \end{aligned}$$

And this being so, starting from the equation

$$x^4 = px^3 - \quad qx^2 + \quad rx - s,$$

that is,

$$= h_1 x^3 - h_0 qx^2 + h_0 rx - h_0 s,$$

we obtain successively

$$\begin{aligned} x^5 = {} & h_1(px^3 - qx^2 + rx - s) \\ & - h_0qx^3 + h_0rx^2 - h_0sx \\ = {} & h_2x^3 - (h_1q - h_0r)x^2 + (h_1r - h_0s)x - h_1s, \end{aligned}$$

$$\begin{aligned} x^6 = {} & h_2(px^3 - qx^2 + rx - s) \\ & - (h_1q - h_0r)x^3 + (h_1r - h_0s)x^2 - h_1sx \\ = {} & h_3x^3 - (h_2q - h_1r + h_0s)x^2 + (h_2r - h_1s)x - h_2s, \end{aligned}$$

$$\begin{aligned} x^7 = {} & h_3(px^3 - qx^2 + rx - s) \\ & - (h_2q - h_1r + h_0s)x^3 + (h_2r - h_1s)x^2 - h_2sx \\ = {} & h_4x^3 - (h_3q - h_2r + h_1s)x^2 + (h_3r - h_2s)x - h_3s, \end{aligned}$$

and so on, the characteristic feature being that by the introduction of the symbols h, the coefficient of x^3 presents itself at each step as a monomial, and the coefficients of the lower powers require no reduction. It is obvious that the process is a perfectly general one, and that for the equation

$$x^m - p_1x^{m-1} + p_2x^{m-2} - \ldots + (-)^m p_m = 0,$$

the formula is

$$\begin{aligned} x^{m+\theta} = {} & \qquad h_{\theta+1}\, x^{m-1} \\ & - \quad (h_\theta p_2 - h_{\theta-1}p_3 + \ldots\ )\, x^{m-2} \\ & + \quad (h_\theta p_3 - h_{\theta-1}p_4 + \ldots\ )\, x^{m-3} \\ & \qquad \vdots \\ & + (-)^{s-1}\, (h_\theta p_s - h_{\theta-1}p_{s+1} + \ldots)\, x^{m-s} \\ & \qquad \vdots \\ & + (-)^{m-1} . h_\theta p_m \qquad x^0, \end{aligned}$$

where, as regards each power of x, the series forming the coefficient thereof is continued as far as possible, that is, up to the term which contains p_m or h_0 as the case may be.