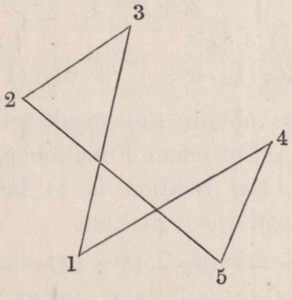
807.

A PROOF OF WILSON'S THEOREM.

[From the *Messenger of Mathematics*, vol. XII. (1883), p. 41.]

LET n be a prime number; and imagine n points, the vertices of a regular polygon; any polygon which can be formed with these n points as vertices is either regular or else it is one of a set of n equal and similar polygons. For instance, $n=5$, the polygon as shown in the figure is one of a set of 5 equal and similar



polygons; in fact, if the points taken in their cyclical order, but beginning at pleasure with any one of the 5 points are called 1, 2, 3, 4, 5, then we have 5 such polygons 13254; and so in general. The whole number of polygons is $\frac{1}{2} . 1 . 2 . 3 \ldots (n-1)$; and the number of the regular polygons is $\frac{1}{2}(n-1)$; hence the number of the remaining polygons is $=\frac{1}{2}(n-1)\{1 . 2 \ldots (n-2)-1\}$; and this number must therefore be divisible by n; that is, $1 . 2 \ldots (n-1)-n+1$ is divisible by n; or, what is the same thing, $1 . 2 \ldots (n-1)+1$ is divisible by n, which is the theorem in question.