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[NOTE ON MR GRIFFITHS' PAPER "ON A DEDUCTION FROM THE ELLIPTIC-INTEGRAL FORMULA $y = \sin(A+B+C+\dots)$".]

[From the *Proceedings of the London Mathematical Society*, vol. XV. (1884), p. 81.]

CONSIDER, for instance,

the cubic transformation

$$y = \frac{x[1+2\alpha' - (1+\alpha')^2 x^2]}{1-\alpha^2 x^2},$$

where $\alpha^2 + \alpha'^2 = 1$.

This implies

$$\sqrt{1-y^2} = \frac{\sqrt{1-x^2}\,[1-(1+\alpha')^2 x^2]}{1-\alpha^2x^2},$$

viz., $\sqrt{1-y^2}$ = a rational multiple of $\sqrt{1-x^2}$.

Also the quadric transformation

$$z = \frac{1-(1+\beta'^2)\,x^2}{1-\beta^2x^2};$$

where $\beta^2 + \beta'^2 = 1$.

This implies

$$\sqrt{1-z^2} = \frac{\sqrt{1-x^2}\,.\,2\beta' x}{1-\beta^2x^2},$$

viz., $\sqrt{1-z^2}$ = a rational multiple of $\sqrt{1-x^2}$.

Hence, assuming

$$u = yz - \sqrt{1-y^2}\,\sqrt{1-z^2},$$

which is a rational function

$$= \frac{x(a_0 - a_2x^2 + a_4x^4)}{1-\alpha^2x^2\,.\,1-\beta^2y^2},$$

we have

$$\sqrt{1-u^2} = y\sqrt{1-z^2} + z\sqrt{1-y^2},$$

which is $= \sqrt{1-x^2}$ multiplied by a like rational function.

That is, in defining the a_0, a_2, a_4, functions of the two arbitrary coefficients α, β, as above, we have in effect so determined them that $\sqrt{1-u^2}$ shall be $=\sqrt{1-x^2}$ multiplied by a rational function of x.

We can then further determine a_0, a_2, a_4 in such wise that the change of x into $\frac{1}{kx}$ shall change u into $\frac{1}{\lambda u}$; and, this being so, making the change in $\sqrt{1-u^2}$, we obtain $\sqrt{1-\lambda^2u^2}$ in the form, $\sqrt{1-k^2x^2}$ multiplied by a rational function of x; viz. u is a function of x such that

$$\frac{du}{\sqrt{1-u^2 \,.\, 1-\lambda^2u^2}} = \frac{Mdu}{\sqrt{1-x^2 \,.\, 1-k^2x^2}}.$$

The theory is thus in effect Jacobi's—with the *novelty* of combining two lower transformations in such wise that the assumed expression for u as a rational function of x shall give

$\sqrt{1-u^2}=\sqrt{1-x^2}$ multiplied by a rational function of x.

It is not necessary that the equations

$y =$ rational function of x and $z =$ rational function of x

should be elliptic-function transformations. All that is required is that they should be such as to give $\sqrt{1-y^2}$ and $\sqrt{1-z^2}$ each $=\sqrt{1-x^2}$ multiplied by a rational function of x.