844.

ON A THEOREM RELATING TO SEMINVARIANTS.

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I FIND among my papers the following example of a general theorem relating to seminvariants.

Write

$$\Theta = b\partial_a + c\partial_b + \ d\partial_c + \ e\partial_d + \ f\partial_e + \ g\partial_f + \ h\partial_g + \ i\partial_h,$$
$$\Omega = \qquad c\partial_b + 2d\partial_c + 3e\partial_d + 4f\partial_e + 5g\partial_f + 6h\partial_g + 7i\partial_h,$$

then from a seminvariant S of the degree δ and weight w, operating upon it with $\Theta - \delta b$ and $\Omega - 2wb$, we derive two seminvariants, each of them of the degree $\delta + 1$ and weight $w + 1$; we obtain a combination of these by operating on S with $2w(\Theta - \delta b) - \delta(\Omega - 2wb)$, that is, with $2w\Theta - \delta\Omega$, and this combination is of the weight $w + 1$, but only of the degree δ; viz. operating with $2w\Theta - \delta\Omega$, we obtain a new seminvariant of the same degree but of a weight increased by unity.

The example relates to the under-mentioned seminvariant S of the degree 3 and weight 7:

	$S=$
h	+ 1
bg	− 7
cf	+ 9
de	− 5
b^2f	+ 12
bce	− 30
bd^2	+ 20
	± 42

	ΘS	$-3bS$	Sum	ΩS	$-14bS$	Sum	$14\Theta S$	$-3\Omega S$			$\div 110$
i	1		1	+ 7		+ 7	+ 14	− 21	+ 7	0	
bh	− 5	− 3	− 8	− 42	− 14	− 56	− 70	+ 126	− 56	0	
cg	+ 2		+ 2	+ 38		+ 38	+ 28	− 114	+ 196	+ 110	+ 1
df	+ 4		+ 4	− 2		− 2	+ 56	+ 6	− 392	− 330	− 3
e^2	− 5		− 5	− 15		− 15	− 70	+ 45	+ 245	+ 220	+ 2
b^2g	+ 5	+ 21	+ 26	+ 60	+ 98	+ 158	+ 70	− 180		− 110	− 1
bcf	+ 3	− 27	− 24	− 96	− 126	− 222	+ 42	+ 288		+ 330	+ 3
bde	+ 5	+ 15	+ 20	+ 60	+ 70	+ 130	+ 70	− 180		− 110	− 1
c^2e	− 30		− 30	− 30		− 30	− 420	+ 90		− 330	− 3
cd^2	+ 20		+ 20	+ 20		+ 20	+ 280	− 60		+ 220	+ 2
b^3f		− 36	− 36		− 168	− 168					
bce		+ 90	+ 90		+ 420	+ 420					
b^2d^2		− 60	− 60		− 280	− 280					
	± 40	± 126	± 163	± 185	± 588	± 773	± 560	± 555	± 448	± 880	± 8

The columns show ΘS (where observe that, to operate with the $b\partial_a$ of Θ, we restore the proper power of a, reading S as being $= +1a^2h - 7abg + \&c.$, and putting ultimately $a = 1$) and $-3bS$, and the sum of these, which is a seminvariant, degree 4, weight 8; also ΩS and $-14bS$, and the sum of these, which is a seminvariant, degree 4, weight 8. They also show $14\Theta S$ and $-3\Omega S$, the sum of which would be a seminvariant, degree 3, and weight 8; instead of giving this sum, I have added a column equal to $+7(i - 8bh + 28cg - 56df + 35e^2)$, and given the sum of the three columns which will of course be a seminvariant of the same degree and weight; the coefficients contain all of them the factor 110, and, throwing this out, we have in the last column the seminvariant $cg - 3df + \ldots + 2cd^2$ of the degree 3 and weight 8, derived by the foregoing direct process from the given seminvariant S of the degree 3 and weight 7.