637.

ON A DIFFERENTIAL EQUATION IN THE THEORY OF ELLIPTIC FUNCTIONS.

[From the *Messenger of Mathematics*, vol. VI. (1877), p. 29.]

In the differential equation

$$Q^2 - Q\left(k + \frac{1}{k}\right) - 3 = 3(1-k^2)\frac{dQ}{dk},$$

considered *Messenger*, t. IV., pp. 69 and 110, [594] and [597], writing $Q = x$ and $k + \frac{1}{k} = y$, the equation becomes

$$dy = \frac{3(y^2-4)\,dx}{3 + xy - x^2},$$

and we have, as a particular solution,

$$y = \tfrac{1}{4}\left(x^3 - 6x - \frac{3}{x}\right).$$

To verify this, observe that from the value of y

$$dy = \frac{3}{4x^2}(x^2-1)^2\,dx, \quad 3 + xy - x^2 = \tfrac{1}{4}(x^2-1)(x^2-9),$$

and the equation becomes

$$\frac{3}{4x^2}(x^2-1)^2 = \frac{\frac{3}{16x^2}\{(x^4-6x^2-3)^2 - 64x^2\}}{\frac{1}{4}(x^2-1)(x^2-9)},$$

viz. this is

$$(x^2-1)^3(x^2-9) = (x^4-6x^2-3)^2 - 64x^2,$$

which is right.