644.

NOTE ON MAGIC SQUARES.

[From the *Messenger of Mathematics*, vol. VI. (1877), p. 168.]

In a magic square of any odd order, formed according to the ordinary process, there is a tolerably simple analytical expression for the number which occupies any given compartment; thus taking the square of 21, let the dexter diagonals (N.W. to S.E.) commencing from the N.E. corner compartment, be numbered 1, 2, 3,.., 20, 21, 20′, 19′,.., 2′, 1′, the diagonals of course containing these numbers of compartments respectively; and in any diagonal let the compartments reckoning from the top line be numbered 1, 2, 3,.., respectively; then if $D_{\theta,\phi}$ (or $D'_{\theta,\phi}$ as the case may be) denotes the number in the compartment ϕ of the diagonal θ or θ', we have

$$\begin{aligned}
D_{2\theta+1,\phi} &= 20\theta + 10 + \phi,\\
D_{2\theta,\phi} &= 20\theta + 231 + \phi\,(-21),\\
D'_{2\theta+1,\phi} &= -22\theta + 430 + \phi,\\
D'_{2\theta,\phi} &= -22\theta + 231 + \phi\,(-21),
\end{aligned}$$

where in the second and fourth expressions the term -21 is to be retained only if $\phi > \theta$; if $\phi \ngtr \theta$, it is to be omitted. There would be a like formulæ for a square of any odd order, and it would be easy to write down the formulæ for the general value $2n+1$: but I have preferred to give them for a specific case.