648.

ALGEBRAICAL THEOREM.

[From the *Quarterly Journal of Pure and Applied Mathematics*, vol. XIV. (1877), p. 53.]

I WISH to put on record the following theorem, given by me as a Senate-House Problem, January, 1851.

If $\{\alpha+\beta+\gamma+\ldots\}^p$ denote the expansion of $(\alpha+\beta+\gamma+\ldots)^p$, retaining those terms $N\alpha^a\beta^b\gamma^c\ldots$ only in which

$$b+c+d+\ldots \not> p-1, \quad c+d+\ldots \not> p-2, \text{ \&c., \&c.,}$$

then

$$x^n = (x+\alpha)^n - n\,\{\alpha\}^1 (x+\alpha+\beta)^{n-1} + \tfrac{1}{2}n\,(n-1)\,\{\alpha+\beta\}^2 (x+\alpha+\beta+\gamma)^{n-2}$$
$$- \tfrac{1}{6}n\,(n-1)\,(n-2)\,\{\alpha+\beta+\gamma\}^3 (x+\alpha+\beta+\gamma+\delta)^{n-3} + \text{ \&c.}$$

The theorem, in a somewhat different and imperfectly stated form, is given, Burg, *Crelle*, t. I. (1826), p. 368, as a generalisation of Abel's theorem,

$$(x+\alpha)^n = x^n + n\alpha\,(x+\beta)^{n-1} + \tfrac{1}{2}n\,(n-1)\,\alpha\,(\alpha-2\beta)\,(x+2\beta)^{n-2}$$
$$+ \tfrac{1}{6}\,(n-1)\,(n-2)\,(n-3)\,\alpha\,(\alpha-3\beta)^2\,(x+3\beta)^2 + \text{ \&c.}$$