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NOTE ON THE THEORY OF CORRESPONDENCE.

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IF the point P on a given curve U of the order m, and the point Q on a given curve V of the order m', have a (1, 1) correspondence, this implying that the two curves have the same deficiency; then if PQ intersects the consecutive line $P'Q'$ in a point R, the locus of R is a curve W of the class $m+m'$, and the point R on this curve has, in general (but not universally), a (1, 1) correspondence with the point P on U or with the point Q on V. For, considering the correspondence of the points P and R, to a given position of P there corresponds, it is clear, a single position of R; on the other hand, starting from R, the tangent at this point to the curve W meets the curve U in m points and the curve V in m' points, but it is in general only one of the m points and only one of the m' points which are corresponding points on the curves U and V; that is, it is only one of the m points which is a point P; and the correspondence of (P, R) is thus a (1, 1) correspondence.

But the curves U, V may be such that the correspondence of (P, R) is not a (1, 1) but a $(k, 1)$ correspondence; viz., that to a given position of P there corresponds a single position of R, but to a given position of R, k positions of P. To show that this is so, imagine through P a line Π having therewith a $(k, 1)$ correspondence; P being, as above, a point on the curve U, the line in question envelopes a curve W; and the correspondence is such that, for any given position of P on the curve U, we have through it a single position of the line: but, for a given tangent of the curve W, we have upon it k positions of the point P, viz. k of the m intersections of the line with the curve U are points corresponding to the line; this, of course, implies that the curve U is not any curve whatever of the order m, but a curve of a peculiar nature.

Imagine now that we have on the line Π a point Q, having with P a (1, 1) correspondence of a given nature: to fix the ideas, suppose P, Q are harmonics in regard to a given conic: since on each of the lines Π there are k positions of P, there are also on the line k positions of Q, and the locus of these k points Q is a curve V, say of the order m'.

The point P on the curve U and the point Q on the curve V have a (1, 1) correspondence. For, consider P as given: there is a single position of the line Π intersecting V in m' points, but obviously only one of these is the point Q. And consider Q as given: then through Q we have say μ tangents of the curve W; each of these tangents intersects the curve U in m points, k of which are points P, but for a tangent taken at random no one of these is the correspondent of Q; it is, in general, only one of the μ tangents which has upon it k points P, one of them being the point corresponding to Q; that is, to a given position of Q there corresponds a single position of P; and the correspondence of the points (P, Q) is thus a (1, 1) correspondence.

We have thus the point P on the curve U and the point Q on the curve V, which points have with each other a (1, 1) correspondence; and the line Π is the line PQ joining these points; this intersects the consecutive line in a point R; and the locus of R is the curve W. To a given position of P there corresponds a single line Π, and therefore a single position of R; but to a given position of R there correspond k positions of P, viz. drawing at R the tangent to the curve W, this is a line Π having upon it k points P, or the correspondence of (P, Q) is, as stated, a $(k, 1)$ correspondence.

The foregoing considerations were suggested to me by the theory of parallel curves. Take a curve parallel to a given curve, for example, the ellipse; this is a curve of the order δ, such that every normal thereto is a normal at two distinct points; and the curve has as its evolute the evolute of the ellipse, *or, more accurately, the evolute of the ellipse taken twice;* but, attending only to the evolute taken once, each tangent of the evolute is a normal of the parallel curve at two distinct points thereof, and the points of the parallel curve have with those of the evolute not a (1, 1) but a (2, 1) correspondence.