917.

[NOTE ON THE THEORY OF RATIONAL TRANSFORMATION.]

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IN my paper, "Note on the Theory of the Rational Transformation between Two Planes, and on Special Systems of Points," *Proc. Lond. Math. Soc.* t. III. (1870), pp. 196—198, [450], I notice a difficulty which presents itself in the theory. The transformation is given by the equations

$$x' : y' : z' = X : Y : Z,$$

where X, Y, Z are functions $(*\between x, y, z)^n$, such that $X=0$, $Y=0$, $Z=0$ are curves in the first plane passing through α_1 points each once, α_2 points each twice (that is, having each of the α_2 points for a double point), α_3 points each 3 times, and so on. We have as the condition of a single variable point of intersection,

$$\alpha_1 + 4\alpha_2 + 9\alpha_3 + \ldots = n^2 - 1,$$

and as the condition in order that each of the curves $X=0$, $Y=0$, $Z=0$, or say the curve $aX+bY+cZ=0$, may be unicursal,

$$\alpha_2 + 3\alpha_3 + \ldots = \tfrac{1}{2}(n-1)(n-2);$$

and we thence deduce

$$\alpha_1 + 3\alpha_2 + 6\alpha_3 + \ldots = \tfrac{1}{2}n(n+3) - 2;$$

viz. the postulation of the fixed points *quoad* a curve of the order n is less by 2 than the postulandum (or, as I prefer to call it, the capacity) $\tfrac{1}{2}n(n+3)$ of the curve of the order n; that is, there are precisely the three asyzygetic curves $X=0$, $Y=0$, $Z=0$. This is as it should be, assuming that the $(\alpha_1, \alpha_2, \alpha_3, \ldots)$ points are an ordinary system of points: but what if they form a special system having a postulation less than $\alpha_1 + 3\alpha_2 + 6\alpha_3 + \dots$? If, for instance, the postulation is $= \alpha_1 + 3\alpha_2 + 6\alpha_3 + \dots - 1$, then this would be $= \frac{1}{2}n(n+3) - 3$, and there would be four asyzygetic curves $X = 0$, $Y = 0$, $Z = 0$, $W = 0$. I believe this to be impossible; but the only proof which I can offer rests upon a remark in regard to the form of the tables*, pp. 148, 149, of my paper "On the Rational Transformation between Two Spaces," *Proc. Lond. Math. Soc.*, t. III. (1870), pp. 127—180, [447]. I recall that the Jacobian curve $J(X, Y, Z) = 0$ consists of α_1' lines, α_2' conics, α_3' cubics, ..., &c., each passing a certain number of times through the $(\alpha_1, \alpha_2, \alpha_3, \dots)$ points, and that the number of times of passage is shown by these tables; thus, *loc. cit.*, $n = 5$, $\alpha_1 = 8$, $\alpha_4 = 1$: the Jacobian consists of eight lines and a quartic, and we have the table $(\alpha_1' = 8, \alpha_4' = 1)$,

	$\alpha_1 = 8$	$\alpha_2 = 0$	$\alpha_3 = 0$	$\alpha_4 = 1$
$\alpha_1' = 8$	1			1
$\alpha_2' = 0$				
$\alpha_3' = 0$				
$\alpha_4' = 1$	8			1^3

showing that the quartic passes through the eight points α_1, and through the point α_4 three times (has α_4 for a triple point). Imagine a new function W. Then in like manner $J(X, Y, W) = 0$ consists of eight lines and a quartic, and this quartic passes through the eight points α_1 and the point α_4 three times; that is, the two quartics intersect in $8 + 3 \cdot 3, = 17$ points; and thus the two quartics must be one and the same curve; this implies a syzygy between X, Y, Z, W, viz. W is a mere linear function of X, Y, Z. The general remark is that, if in the tables m^p is reckoned as mp^2, then in the table for the several lines (exclusive of those for which the outside accented letter is $= 0$, and therefore the tabular numbers of the line are each $= 0$), i.e. for the lines which correspond to a line, a conic, a cubic, a quartic, &c., respectively, the sums of the tabular numbers are $1^2 + 1$, $2^2 + 1$, $3^2 + 1$, $4^2 + 1$, &c., respectively. This is, in fact, the case for each of the eleven tables (*loc. cit.*).

[* This Collection, vol. VII., pp. 208, 209.]