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ON A PROBLEM IN THE CALCULUS OF VARIATIONS.

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THE problem is, $z = \frac{1}{3}(3x - y^2)y$, to find y a function of x such that $\int zdx =$ max. or min., subject to a given condition $\int ydx = c$ (the limits of each integral being x_1, x_0, where these quantities are each positive, and $x_1 > x_0$). The ordinary method of solution gives $y^2 = x + \lambda$, where $(x_1 + \lambda)^{\frac{3}{2}} - (x_0 + \lambda)^{\frac{3}{2}} = \frac{3}{2}c$; so long as c is not less than $(x_1 - x_0)^{\frac{3}{2}}$, there is a real value of λ, but for a smaller value of c there is no real value. The difficulty arising in this last case is somewhat illustrated by replacing the original problem by a like problem of ordinary maxima and minima; viz., $x_1, x_2 \ldots x_n$ being given positive values of x, in the order of increasing magnitude; and if, in general, $z_i = (3x_i - y_i^2)y_i$, then the problem is to find y_i a function of x_i, such that $\Sigma z_i =$ max. or min., subject to the condition $\Sigma y_i = c$. We have here $y_i^2 = x_i + \lambda$, where λ is to be determined by the condition $\Sigma y_i = c$; the remainder of the investigation turns on the question of the sign $y_i = +\sqrt{x_i + \lambda}$ or $y_i = -\sqrt{x_i + \lambda}$, to be taken for the several values of i respectively.