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ON THE ANHARMONIC-RATIO SEXTIC.

[From the *Quarterly Journal of Pure and Applied Mathematics*, vol. x. (1870), pp. 56, 57.]

Mr Walker's equation is $\Delta(\lambda^2-\lambda+1)^3+I^3(\lambda^2-\lambda)^2=0$; changing the sign of λ, and also the numerical multipliers of I, Δ (so as to convert the discriminant equation into its standard form $\Delta=I^3-27J^2$), the equation is

$$4\Delta(\lambda^2+\lambda+1)^3-27I^3(\lambda^2+\lambda)^2=0.$$

I remark that this is most readily obtained as follows; writing

$$A=(a-d)(b-c),$$
$$B=(b-d)(c-a),$$
$$C=(c-d)(a-b),$$

then we have $A+B+C=0$,

$$I=\tfrac{1}{24}(A^2+B^2+C^2)=-\tfrac{1}{12}(BC+CA+AB),$$
$$J=\tfrac{1}{432}(B-C)(C-A)(A-B),$$
$$\sqrt{(\Delta)}=\tfrac{1}{16}ABC,$$

see my Fifth Memoir on Quantics, *Phil. Trans.*, vol. CXLVIII. (1858), pp. 429—460, [156]. And observe also, that in virtue of the relation $A+B+C=0$, we have

$$12I=A^2+AB+B^2=A^2+AC+C^2=B^2+BC+C^2.$$

Hence writing

$$u=\frac{4\sqrt{(\Delta)}}{3I}\left(\lambda+\frac{1}{\lambda}+1\right),$$

when λ has any one of the values $\frac{A}{B}, \frac{B}{A}, \frac{A}{C}, \frac{C}{A}, \frac{B}{C}, \frac{C}{B}$, we see that u assumes only the values A, B, C, and u is thus determined by the equation

$$u^3 - 12Iu - 16\sqrt{(\Delta)} = 0.$$

Eliminating u, we obtain

$$16\sqrt{(\Delta)}\left\{\frac{4\Delta}{27I^3}\left(\lambda + \frac{1}{\lambda} + 1\right)^3 - \left(\lambda + \frac{1}{\lambda} + 1\right) - 1\right\} = 0,$$

or, what is the same thing,

$$4\Delta\left(\lambda + \frac{1}{\lambda} + 1\right)^3 - 27I^3\left(\lambda + \frac{1}{\lambda} + 2\right) = 0,$$

that is

$$4\Delta(\lambda^2 + \lambda + 1)^3 - 27I^3\lambda^2(\lambda + 1)^2 = 0,$$

the required equation.