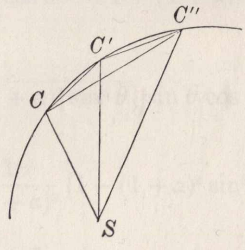
471.

NOTE ON THE PROBLEM OF THE DETERMINATION OF A PLANET'S ORBIT FROM THREE OBSERVATIONS.

[From the *Monthly Notices of the Royal Astronomical Society*, vol. XXIX. (1868—1869), pp. 257—259.]

THE principle of the solution given in the *Theoria Motus* may be explained very simply as follows:

Consider three successive positions of C, C', C'', of a planet revolving about the focus S; let n, n', n'', denote the doubles of the triangular areas $C'SC''$, CSC', and CSC'' respectively (viz. the triangular area means the area of the triangle included between the two radius vectors and the chord joining their extremities), r' the radius



vector SC'; θ'', θ, the times of describing the arcs CC' and $C'C''$ respectively, the units of time and distance being such that the time is equal to the double area divided by the square root of the half latus rectum ($t = 2\pi a^{\frac{3}{2}}$ for the Period in a circular or elliptic orbit).

Then writing

$$P = \frac{n''}{n}, \quad Q = 2\left(\frac{n+n''}{n'} - 1\right) r'^3,$$

(observe that $n+n''-n'$ is $=$ twice the triangle $CC'C''$), for neighbouring positions of the planet, the values of P and Q are *approximately* $=\frac{\theta''}{\theta}$ and $\theta\theta''$ respectively: the solution consists in the determination of an orbit for which P and Q have these approximate values; then, by means of such approximate orbit, the values of P and Q are more accurately determined, and by means of these new values of P and Q, a new determination is effected of the orbit: and so on, to the requisite accuracy of approximation.

The foregoing approximate values of P and Q respectively are deduced from the accurate values

$$P=\frac{\theta''\eta}{\theta\eta''},\quad Q=\frac{\theta\theta''}{\eta\eta''}\frac{r'^2}{r'r''}\frac{1}{\cos f\cos f'\cos f''};$$

where r, r', r'' are the radius vectors SC, SC', SC''; $2f$, $2f'$, $2f''$ are the angular distances $C'SC''$, CSC'', CSC' $(f'=f+f'')$ and η, η', η'' are the ratios of the sectorial areas $C'SC''$, CSC'', $C'SC''$, to the triangular areas represented by the same letters respectively: the doubles of the sectorial areas are thus $n\eta$, $n'\eta'$, and $n''\eta''$, and if the half latus rectum be denoted by p, then we have

$$\sqrt{p}=\frac{n\eta}{\theta}=\frac{n'\eta'}{\theta'}=\frac{n''\eta''}{\theta''};$$

and it thus at once appears that the accurate value of P is $=\frac{\theta''\eta}{\theta\eta''}$, as above. To obtain the expression for Q, taking ϕ, ϕ', ϕ'' for the true anomalies (and, for greater symmetry, writing for the moment ν, $-\nu'$, ν'', g, $-g'$, g'' in place of n, n', n'', f, f', f'' respectively), we have

$$r=\frac{p}{1+e\cos\phi},\quad 2g=\phi''-\phi',$$

$$r'=\frac{p}{1+e\cos\phi'},\quad 2g'=\phi-\phi'',$$

$$r''=\frac{p}{1+e\cos\phi''},\quad 2g''=\phi'-\phi,$$

$$(g+g'+g''=0);$$

whence identically

$$\frac{\sin 2g}{r}+\frac{\sin 2g'}{r'}+\frac{\sin 2g''}{r''}=-\frac{4\sin g\sin g'\sin g''}{p};$$

or writing

$$\nu=r'r''\sin 2g,\quad \nu'=r''r\sin 2g',\quad \nu''=rr'\sin 2g'',$$

this is

$$\nu+\nu'+\nu''=-\frac{4rr'r''\sin g\sin g'\sin g''}{p},$$

$$=-\frac{(rr'r'')^2\sin 2g\sin 2g'\sin 2g''}{2prr'r''\cos g\cos g'\cos g''}$$

$$=-\frac{\nu\nu'\nu''}{2prr'r''\cos g\cos g'\cos g''}.$$

www.rcin.org.pl

This is, in fact,

$$n - n' + n'' = \frac{nn'n''}{2prr'r'' \cos f \cos f' \cos f''},$$

or since

$$\frac{nn''}{p} = \frac{\theta\theta'}{\eta\eta''},$$

it is

$$2\left(\frac{n+n''}{n'} - 1\right) = \frac{\theta\theta''}{\eta\eta'' rr'r'' \cos f \cos f' \cos f''},$$

viz. multiplying by r'^3, it is

$$Q \quad = \frac{\theta\theta''}{\eta\eta''} \frac{r'^2}{rr''} \frac{1}{\cos f \cos f' \cos f''},$$

the above-mentioned value of Q.

www.rcin.org.pl