NOTES AND REFERENCES.

445, 451, 454. WE have the two papers by K. Rohn, "Die Flächen vierter Ordnung hinsichtlich ihrer Knotenpunkte und ihrer Gestalten," *Preisschr. der F. J. Gesell. zu Leipzig* (Leipzig, 1886, pp. 1—58), and same title *Math. Ann.* t. XXIX. (1887), pp. 81—97. I have not been able to examine the conclusions arrived at in these papers with as much care as would have been desirable.

I call to mind that for a k-nodal quartic surface the tangent cone from any node is a sextic cone with $(k-1)$ nodal lines, breaking up it may be into cones of lower orders—see table p. 265: and that we distinguish the quartic surfaces according to the forms of the sextic cones corresponding to the k nodes respectively. It will be recollected that (6) denotes a sextic cone, (6_1) a sextic cone with one nodal line, $(5_1, 1)$ a sextic cone breaking up into a quintic cone with one nodal line and a plane; and so in other cases.

There is a sort of break in the theory; in fact when the number of nodes is not greater than 7 these may be any given points whatever, and taking the 7 points at pleasure we have surfaces with 8 nodes, and 9 nodes, but not with any greater number of nodes, viz. for a surface with 10 or more nodes, it is not permissible to take 7 of these as points at pleasure, so that the theory of the surfaces with 10 or more nodes is so to speak separated off from that of the surfaces with a smaller number of nodes. For the case of 10 nodes we have the symmetroid $10(3, 3)$ and other forms, for 11 nodes Rohn finds 3 or ? 4 forms; for 12 nodes he has four forms, viz. my 3 forms and a fourth form 12_d; for 13 nodes he has two forms, 13_b agreeing with my 13_α, and 13_a which replaces my non-existent form 13_β; for 14 nodes, 15 nodes and 16 nodes he has in each case a single form, agreeing with my results. Without endeavouring to complete the theory, I write down a table as follows:

No. of Nodes		Form of Cones.	Remarks
16		$16(1, 1, 1, 1, 1, 1)$	
15		$15(2, 1, 1, 1, 1)$	
14		$8(3_1, 1, 1, 1)+6(2, 2, 1, 1)$	
13	$13_b=13_\alpha$	$3(4_3, 1, 1)+1(3, 1, 1, 1)+9(3_1, 2, 1)$	
,,	13_a	$1(2, 2, 2)+12(4_3, 1, 1)$	13_a replaces my non-existent 13_β, $=13(2, 2, 2)$
12	$12_b=12_\alpha$	$12(4_3, 2)$	
,,	$12_a=12_\beta$	$2(5_6, 1)+6(3_1, 3_1)+4(3, 2, 1)$	

Table continued.

No. of Nodes	Form of Cones		Remarks
12	$12_c = 12_\gamma$	$12(4_2,\ 1,\ 1)$	$12_c = 12_\gamma$ is a peculiarly simple and elegant form; the equation is $A^2 - xyzw = 0$, where A is a quadric function of the coordinates.
,,	12_d	$2(4_2,\ 1,\ 1) + 8(5_6,\ 1) + 2(4_3,\ 2)$	
11	$11_a = 11_\alpha$	$1(6_{10}) + 10(3_1,\ 3_1)$	
,,	11_b	$8(6_{10}) + 3(4_2,\ 2)$	
,,	11_c	$6(5_5,\ 1) + 5(6_{10})$	
,,	11_d	?	
10		$10(3,\ 3)$	The quartic surface is here the symmetroid.
9			
8			
7			
6			
5		$5(6_4)$	
4		$4(6_3)$	
3		$3(6_2)$	
2		$2(6_1)$	
1		$1(6)$	

The suffixes a, b, c, d refer to Rohn's forms, the suffixes α, β, γ to my forms. The form 11_d is given in the first but not in the second of Rohn's two memoirs, and I am not sure as to the intended character of the sextic cones. I have not attempted to fill up the third column of the table for the Nos. of nodes 9, 8, 7, 6, as there may be particular cases which I have not considered. For the Nos. 5, 4, 3, 2, 1, the cone is a sextic cone with at most 4 nodal lines, and consequently in each case a proper sextic cone not breaking up into cones of inferior orders.

END OF VOL. VII.

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