

11.371/3

MATHEMATICAL
PAPERS

—
SILVESTER

VOL. III



CAMBRIDGE
UNIVERSITY PRESS

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THE COLLECTED
MATHEMATICAL PAPERS

OF
JAMES JOSEPH SYLVESTER

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OF THE UNIVERSITY OF CAMBRIDGE

CAMBRIDGE UNIVERSITY PRESS

100 Brook Hill Drive

Cambridge, Massachusetts 02142

MATHEMATICAL PAPERS

PRINTED AND BOUND BY

THE UNIVERSITY PRESS

CAMBRIDGE

AND THE UNIVERSITY PRESS

OF THE STRAITS SETTLEMENTS

Cambridge

at the University Press

100 Brook Hill Drive

CAMBRIDGE UNIVERSITY PRESS

London: FETTER LANE, E.C.

C. F. CLAY, MANAGER



Edinburgh: 100, PRINCES STREET

Berlin: A. ASHER AND CO.

Leipzig: F. A. BROCKHAUS

New York: G. P. PUTNAM'S SONS

Bombay and Calcutta: MACMILLAN AND CO., LTD.

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MATHEMATICAL PAPERS

OF

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and Savilian Professor in the University of Oxford

VOLUME III

(1870—1883)



Cambridge

At the University Press

1909

opis nr: 56611

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and various Professor in the University of Oxford.

Cambridge :

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11.371 / III

Dr. D.

K 208 / 55

PREFATORY NOTE.

THE present volume deals very largely with the Author's enumerative method of obtaining the complete system of concomitants of a system of quantics, with the help of generating functions; the brief but very luminous papers here reprinted, at the end of the volume, from the *Johns Hopkins University Circulars* shew the Author preparing his memoir on the Constructive Theory of Partitions, which begins the next, and last, volume of his Mathematical Works. The previous volume included the period of the Author's activity at the Military Academy, Woolwich; this volume nearly covers the time of that surely most interesting experiment in educational method when, at Baltimore, unhindered by traditional routine, and encouraged to give full rein to his invention, he was able, nay obliged, as he tells us (p. 76), to yield to the inquisitive student who would have the New Algebra, that or nothing; with results that are imperishable. The matter is seen so well from the Author's point of view in his Commemoration day Address at Johns Hopkins University (1877), that, after some hesitation, a reprint of this is included in the present volume (No. 10). The Remarks on Research, in *Nature*, vol. XVI. (1877), are from this Address. The present volume also includes the Author's investigations on Chemistry and Algebra (No. 24), the paper on Certain Ternary Cubic-Form Equations (No. 39), and the paper on Subinvariants and Perpetuants (No. 67). In connection with the enumerative methods in this volume the reader's attention may be directed to a paper, by F. Franklin, "On the Calculation of the Generating Functions and Tables of Groundforms for Binary Quantics," in the *American Journal of Mathematics*, III. (1880), pp. 128—153, to which, as to one or two other memoirs referring to matters dealt with in the text, I have ventured to add a reference at the appropriate place.

H. F. BAKER.

ST JOHN'S COLLEGE, CAMBRIDGE.

24 November 1909.

PRELIMINARY NOTE

The present volume deals very largely with the Author's constructive method of obtaining the complete system of concomitants of a system of quantities with the help of generating functions; the brief but very luminous papers here reprinted at the end of the volume, from the Johns Hopkins University Library, show the Author preparing his memoir on the Constructive Theory of Partitions which begins the next and last volume of his Mathematical Works. The previous volume included the period of the Author's activity at the Military Academy, Warsaw; this volume nearly covers the time of that army's most interesting experiment in educational method when at Baltimore, supervised by traditional routine, and encouraged to give full scope to his invention he was able, nay obliged, as he tells us (p. 10), to yield to the inquisitive student who would have the New Algebra, that of nothing; with results that are unparalleled. The matter is said well from the Author's point of view in his Commemorative day Address at Johns Hopkins University (1877), and after some dedication, a reprint of this is included in the present volume (No. 10). The Remarks on Research in Volume vol. XVI (1877) are from this Address. The present volume also includes the Author's investigations on Chemistry and Algebra (No. 24), the paper on Certain Ternary Cubic Form Equations (No. 23), and the paper on Substitutions and Progressions (No. 17). In connection with the constructive method in this volume the reader's attention may be directed to a paper by F. Franklin, "On the Calculation of the Generating Functions and Tables of Recurrences for Binary Quotients," in the American Journal of Mathematics, III (1880), pp. 122-123, to which, as to one or two other memoirs relating to matters dealt with in the text, I have referred to with a reference at the appropriate place.

H. F. HARKER

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