

TABLES OF GENERATING FUNCTIONS, REDUCED AND REPRESENTATIVE, FOR CERTAIN TERNARY SYSTEMS OF BINARY FORMS.

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THE annexed tables have been calculated under my directions by Messrs Durfee and Ely, out of the fund placed at my disposition by the British Association for the Advancement of Science in the year 1881. Subsequent investigation will be necessary in order to ascertain whether there exist or not extra tabular groundforms which escape the operation of tamisage.

G. F. it will be understood stands for the words Generating Function.

SYSTEM OF TWO QUADRATICS AND ONE QUARTIC.

G. F. for invariants, reduced form.

$$\text{Denominator: } (1 - b^2)(1 - \beta^2)(1 - d^2)(1 - d^3)(1 - b\beta)(1 - bd)(1 - \beta d)(1 - b^2d)(1 - \beta^2d).$$

Numerator:

| | | d^0 | d^1 | d^2 | d^3 | d^4 | | | d^0 | d^1 | d^2 | d^3 | d^4 |
|-------|-----------|-------|----------------|----------------|----------------|-------|-------|-----------|-------|-------|----------------|----------------|----------------|
| | β^0 | 1 | | | | | | β^1 | | | $\overline{1}$ | | |
| b^0 | β^1 | | $\overline{1}$ | | | | b^3 | β^2 | | | | 1 | |
| | β^2 | | | 1 | | | | β^3 | | | | | $\overline{1}$ |
| | β^0 | | $\overline{1}$ | | | | | β^0 | | | 1 | | |
| b^1 | β^1 | | 1 | 2 | | | b^2 | β^1 | | 1 | | $\overline{1}$ | |
| | β^2 | | 1 | | $\overline{1}$ | | | β^2 | | | $\overline{2}$ | $\overline{1}$ | |
| | β^3 | | | $\overline{1}$ | | | | β^3 | | | | 1 | |

G. F. for invariants, representative form.

$$\text{Denominator: } (1 - b^2)(1 - \beta^2)(1 - d^2)(1 - d^3)(1 - b\beta)(1 - b^2d^2)(1 - \beta^2d^2)(1 - b^2d)(1 - \beta^2d).$$

Numerator :

| | | | | | | | |
|-------|-----------|-------|-------|-------|----------------|----------------|----------------|
| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 |
| | β^0 | 1 | | | | | |
| b^0 | β^1 | | | | | | |
| | β^2 | | | | | | |
| | β^3 | | | | 1 | | |
| | β^4 | | | | | | |
| | β^0 | | | | | | |
| | β^1 | | 1 | 1 | | | |
| b^1 | β^2 | | 1 | 1 | 1 | | |
| | β^3 | | | | | | |
| | β^4 | | | | $\overline{1}$ | | |
| | β^0 | | | | | | |
| b^2 | β^1 | | 1 | 1 | 1 | | |
| | β^2 | | | | | | |
| | β^3 | | | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ |
| | β^4 | | | | | | |

| | | | | | | | | |
|-------|-----------|-------|-------|-------|----------------|----------------|----------------|----------------|
| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 |
| | β^0 | | | | | | | |
| b^4 | β^1 | | | | $\overline{1}$ | | | |
| | β^2 | | | | | | | |
| | β^3 | | | | | | | |
| | β^4 | | | | | | | $\overline{1}$ |
| | β^0 | | | | | | | |
| | β^1 | | | | 1 | | | |
| b^3 | β^2 | | | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | |
| | β^3 | | | | | $\overline{1}$ | $\overline{1}$ | |
| | β^4 | | | | | | | |

TABLE OF GROUNDFORMS.

| | Deg. in coeff's of quadratic | Deg. in coeff's of quadratic | Deg. in coeff's of quartic. | | | |
|---|------------------------------------|------------------------------------|--------------------------------|---|---|---|
| | | | 0 | 1 | 2 | 3 |
| 0 | 0 | | | | 1 | 1 |
| | 1 | | | | | |
| | 2 | 1 | 1 | 1 | | |
| | 3 | | | | | 1 |
| 1 | 0 | | | | | |
| | 1 | 1 | 1 | 1 | | |
| | 2 | | 1 | 1 | 1 | |
| 2 | 0 | 1 | 1 | 1 | | |
| | 1 | | 1 | 1 | 1 | |
| 3 | 0 | | | | | 1 |

SYSTEM OF QUADRATIC, CUBIC, AND QUARTIC.

G. F. for invariants, reduced form.

$$\begin{aligned} \text{Denominator: } & (1 - b^2)(1 - c^4)(1 - d^2)(1 - d^3)(1 - bc^2)(1 - b^3c^2) \\ & (1 - bd)(1 - b^2d)(1 - c^2d)(1 - c^2d^3)(1 - c^4d) \\ & (1 - c^4d^3). \end{aligned}$$

Numerator:

| | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | d^9 | d^{10} |
|-----------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| c^0 | 1 | | | | | | | | | | |
| c^2 | | $\overline{1}$ | | | | | | | | | |
| $b^0 c^4$ | | | 2 | 2 | 2 | 1 | | | | | |
| c^6 | | | 1 | 1 | $\overline{1}$ | $\overline{1}$ | | | | | |
| c^8 | | | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{2}$ | | | | |
| c^{10} | | | | | | | | 1 | | | |
| c^{12} | | | | | | | | | $\overline{1}$ | | |
| c^0 | | $\overline{1}$ | | | | | | | | | |
| c^2 | | 2 | 4 | 2 | 1 | | | | | | |
| c^4 | | 2 | 2 | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | | | | |
| $b^1 c^6$ | | | $\overline{2}$ | $\overline{3}$ | $\overline{3}$ | 1 | 1 | 1 | | | |
| c^8 | | | $\overline{1}$ | $\overline{1}$ | | 1 | 1 | 1 | | | |
| c^{10} | | | | 1 | 1 | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | | | |
| c^{12} | | | | | | | $\overline{1}$ | | | | |
| c^{14} | | | | | | | | | 1 | | |
| c^0 | | | 1 | | | | | | | | |
| c^2 | | 2 | 1 | $\overline{1}$ | $\overline{1}$ | | | | | | |
| c^4 | | 1 | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | |
| c^6 | | | $\overline{1}$ | | | 1 | 1 | | | | |
| $b^2 c^8$ | | | $\overline{1}$ | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | | | |
| c^{10} | | | | $\overline{1}$ | $\overline{1}$ | | | 1 | 1 | | |
| c^{12} | | | | | 1 | 2 | 2 | 1 | 2 | 1 | |
| c^{14} | | | | | | | | | | $\overline{1}$ | |
| c^2 | | 1 | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | | |
| c^4 | 1 | 1 | | 1 | | $\overline{1}$ | | | | | |
| c^6 | | $\overline{2}$ | $\overline{3}$ | $\overline{2}$ | $\overline{2}$ | 1 | $\overline{2}$ | $\overline{1}$ | $\overline{1}$ | | |
| $b^3 c^8$ | | | $\overline{1}$ | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | 1 | 2 | 1 | 1 |
| c^{10} | | | 1 | 1 | 2 | 1 | 2 | 2 | 3 | 2 | |
| c^{12} | | | | | 1 | | $\overline{1}$ | | | $\overline{1}$ | $\overline{1}$ |
| c^{14} | | | | | | | | 1 | 1 | 1 | $\overline{1}$ |

| | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | d^9 | d^{10} |
|--------------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| c^4 | | | 1 | | | | | | | | |
| c^6 | | | | $\overline{1}$ | | | | | | | |
| $b^5 c^8$ | | | | | 2 | 2 | 2 | 1 | | | |
| c^{10} | | | | | 1 | 1 | $\overline{1}$ | $\overline{1}$ | | | |
| c^{12} | | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | $\overline{2}$ | | |
| c^{14} | | | | | | | | | | 1 | |
| c^{16} | | | | | | | | | | | $\overline{1}$ |
| c^2 | | | $\overline{1}$ | | | | | | | | |
| c^4 | | $\overline{1}$ | | 1 | | | | | | | |
| c^6 | | | 2 | 2 | 1 | | $\overline{1}$ | $\overline{1}$ | | | |
| $b^3 c^8$ | | | | $\overline{1}$ | $\overline{1}$ | 1 | | 1 | 1 | | |
| c^{10} | | | | $\overline{1}$ | $\overline{1}$ | 1 | 3 | 3 | 2 | | |
| c^{12} | | | | | 1 | 2 | 1 | | $\overline{2}$ | $\overline{2}$ | |
| c^{14} | | | | | | | $\overline{1}$ | $\overline{2}$ | $\overline{4}$ | $\overline{2}$ | |
| c^{16} | | | | | | | | | | | 1 |
| c^2 | | 1 | | | | | | | | | |
| c^4 | | $\overline{1}$ | 2 | 1 | 2 | 2 | 1 | | | | |
| c^6 | | $\overline{1}$ | $\overline{1}$ | | | 1 | 1 | | | | |
| c^8 | | | | | 1 | 2 | 2 | 1 | 1 | | |
| $b^4 c^{10}$ | | | | | $\overline{1}$ | $\overline{1}$ | | | | 1 | |
| c^{12} | | | | | | 1 | 1 | 1 | 1 | $\overline{1}$ | $\overline{1}$ |
| c^{14} | | | | | | | 1 | 1 | 1 | $\overline{1}$ | $\overline{2}$ |
| c^{16} | | | | | | | | | | | $\overline{1}$ |

G. F. for invariants, representative form.

$$\text{Denominator: } (1 - b^2)(1 - c^4)(1 - d^2)(1 - d^3)(1 - bc^2)(1 - b^3c^2)(1 - b^2d^2) \\ (1 - b^2d)(1 - c^4d^2)(1 - c^2d^3)(1 - c^4d)(1 - c^4d^3).$$

Numerator :

| | a^0 | a^1 | a^2 | a^3 | a^4 | a^5 | a^6 | a^7 | a^8 | a^9 | a^{10} | a^{11} |
|-----------|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|----------|----------|
| c^0 | 1 | | | | | | | | | | | |
| c^2 | | | | | | | | | | | | |
| c^4 | | | 1 | 2 | 2 | 1 | | | | | | |
| $b^0 c^6$ | | | 1 | 3 | 2 | 1 | | | | | | |
| c^8 | | | | | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{1}{1}$ | | | | | |
| c^{10} | | | | | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | | |
| c^{12} | | | | | | | | | | | | |
| c^{14} | | | | | | | | | | 1 | | |
| c^2 | | 2 | 3 | 2 | 1 | | | | | | | |
| c^4 | | 2 | 4 | 5 | 3 | 1 | | | | | | |
| c^6 | | | | | | | | | | | | |
| $b^1 c^8$ | | | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | | |
| c^{10} | | | | | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{2}{2}$ | | | | | |
| c^{12} | | | | 1 | 1 | | $\frac{2}{2}$ | $\frac{1}{2}$ | | | | |
| c^{14} | | | | | | | | | | | | |
| c^{16} | | | | | | | | | | 1 | | |
| c^2 | | 2 | 3 | 3 | 1 | | | | | | | |
| c^4 | | 1 | 3 | 4 | 2 | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | |
| c^6 | | | | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | | |
| $b^2 c^8$ | | | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{5}{5}$ | $\frac{5}{2}$ | $\frac{2}{1}$ | $\frac{2}{2}$ | 1 | | | |
| c^{10} | | | | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | | |
| c^{12} | | | | | 1 | 2 | 2 | 1 | | | | |
| c^{14} | | | | | | 1 | 2 | 2 | 1 | 1 | 1 | 1 |
| c^0 | | | | 1 | | | | | | | | |
| c^2 | | 1 | 1 | | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{1}$ | | | | | |
| c^4 | 1 | 1 | 2 | 1 | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{1}{1}$ | | | | | |
| c^6 | | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{2}{2}$ | $\frac{2}{2}$ | $\frac{2}{2}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | |
| $b^3 c^8$ | | | $\frac{1}{3}$ | $\frac{6}{6}$ | $\frac{5}{4}$ | $\frac{4}{2}$ | | | 1 | | | |
| c^{10} | | | | | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | $\frac{1}{4}$ | $\frac{4}{4}$ | $\frac{2}{2}$ | | |
| c^{12} | | | | 1 | 2 | 3 | 2 | 2 | 3 | 4 | 3 | 1 |
| c^{14} | | | | | 1 | 2 | 3 | 2 | | | | |
| c^{16} | | | | | | | | | 1 | 1 | 1 | 1 |

| | a^1 | a^2 | a^3 | a^4 | a^5 | a^6 | a^7 | a^8 | a^9 | a^{10} | a^{11} | a^{12} |
|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| c^4 | | | 1 | | | | | | | | | |
| c^6 | | | | | | | | | | | | |
| c^8 | | | | | 1 | 2 | 2 | 1 | | | | |
| c^{10} | | | | | 1 | 3 | 2 | 1 | | | | |
| $b^7 c^{12}$ | | | | | | | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{3}{3}$ | $\frac{1}{1}$ | | |
| c^{14} | | | | | | | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | |
| c^{16} | | | | | | | | | | | | |
| c^{18} | | | | | | | | | | | | 1 |
| c^2 | | | $\frac{1}{2}$ | | | | | | | | | |
| c^4 | | | | | | | | | | | | |
| c^6 | | | 1 | 2 | 2 | | $\frac{1}{2}$ | $\frac{1}{1}$ | | | | |
| $b^6 c^8$ | | | | 2 | 3 | 2 | 1 | | | | | |
| c^{10} | | | | | 1 | 2 | 3 | 3 | 3 | 1 | | |
| c^{12} | | | | | | | | | | | | |
| c^{14} | | | | | | | $\frac{1}{2}$ | $\frac{3}{3}$ | $\frac{5}{4}$ | $\frac{4}{2}$ | | |
| c^{16} | | | | | | | | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{2}$ | | |
| c^4 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | | |
| c^6 | | | | | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | |
| c^8 | | | | | 1 | 2 | 3 | 2 | 1 | | | |
| $b^5 c^{10}$ | | | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{1}{2}$ | $\frac{2}{2}$ | $\frac{5}{5}$ | $\frac{5}{3}$ | $\frac{3}{1}$ | | | |
| c^{12} | | | | | 1 | 2 | 3 | 2 | 1 | | | |
| c^{14} | | | | | | 1 | 2 | 1 | $\frac{2}{2}$ | $\frac{4}{3}$ | $\frac{3}{1}$ | |
| c^{16} | | | | | | | | | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{2}$ | |
| c^2 | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | | | | | | | |
| c^4 | | | | | $\frac{2}{2}$ | $\frac{3}{2}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | |
| c^6 | $\frac{1}{2}$ | $\frac{3}{3}$ | $\frac{4}{3}$ | $\frac{3}{2}$ | $\frac{2}{2}$ | $\frac{2}{2}$ | $\frac{3}{2}$ | $\frac{2}{1}$ | | | | |
| $b^4 c^8$ | | $\frac{2}{2}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{1}{1}$ | $\frac{1}{1}$ | $\frac{2}{2}$ | $\frac{1}{1}$ | | | | |
| c^{10} | | | | | $\frac{1}{2}$ | | 2 | 4 | 5 | 6 | 3 | 1 |
| c^{12} | | | | | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| c^{14} | | | | | | 1 | 3 | 3 | 1 | $\frac{1}{2}$ | $\frac{1}{1}$ | $\frac{1}{1}$ |
| c^{16} | | | | | | | | 1 | 1 | 1 | | |
| c^{18} | | | | | | | | | | | $\frac{1}{1}$ | |

TABLE OF GROUNDFORMS.

| Deg. in coeff's of Quadratic | Deg. in coeff's of Cubic. | Degree in coeff's of Quartic. | | | | | |
|------------------------------------|---------------------------------|----------------------------------|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 |
| 0 | 0 | | | 1 | 1 | | |
| | 2 | | | | 1 | | |
| | 4 | 1 | 1 | 2 | 3 | 2 | 1 |
| | 6 | | | 1 | 3 | 2 | 1 |
| 1 | 2 | 1 | 2 | 3 | 2 | 1 | |
| | 4 | | 2 | 4 | 5 | 3 | 1 |
| 2 | 0 | 1 | 1 | 1 | | | |
| | 2 | | 2 | 3 | 3 | 1 | |
| | 4 | | 1 | | | | |
| 3 | 0 | | | | 1 | | |
| | 2 | 1 | 1 | 1 | | | |
| | 4 | 1 | 1 | | | | |
| 4 | 2 | | 1 | 1 | | | |

SYSTEM OF ONE QUADRATIC AND TWO QUARTICS.

G. F. for invariants, reduced form.

$$\text{Denominator: } (1 - b^2)(1 - \delta^2)(1 - \delta^3)(1 - d^2)(1 - d^3)(1 - b\delta)(1 - b^2\delta) \\ (1 - bd)(1 - b^2d)(1 - \delta d)(1 - \delta^2d)(1 - \delta d^2).$$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 |
|-------|------------|----------------|----------------|----------------|----------------|----------------|----------------|-------|-------|------------|----------------|----------------|----------------|----------------|----------------|----------------|-------|
| | δ^0 | 1 | | | | | | | | δ^2 | | | 1 | | | | |
| | δ^1 | | | | | | | | | δ^3 | | | | | | | |
| b^0 | δ^2 | | | 1 | | | | | b^5 | δ^4 | | | | 1 | | | |
| | δ^3 | | | | | | | | | δ^5 | | | | | | | |
| | δ^4 | | | | | 1 | | | | δ^6 | | | | | | | 1 |
| | δ^0 | | $\overline{1}$ | | | | | | | δ^1 | | $\overline{1}$ | | | | | |
| | δ^1 | $\overline{1}$ | 1 | 1 | 1 | | | | | δ^2 | $\overline{1}$ | | | | | | |
| b^1 | δ^2 | | 1 | 1 | | | | | b^4 | δ^3 | | | 1 | | 1 | | |
| | δ^3 | | 1 | | 1 | | | | | δ^4 | | | | 1 | 1 | | |
| | δ^4 | | | | | | $\overline{1}$ | | | δ^5 | | | 1 | 1 | 1 | $\overline{1}$ | |
| | δ^5 | | | | | $\overline{1}$ | | | | δ^6 | | | | | $\overline{1}$ | | |
| | δ^0 | | | 1 | | | | | | δ^1 | | 1 | $\overline{1}$ | | | | |
| | δ^1 | | 2 | | $\overline{1}$ | $\overline{1}$ | | | | δ^2 | $\overline{1}$ | | | | | $\overline{1}$ | |
| b^2 | δ^2 | 1 | | $\overline{1}$ | $\overline{2}$ | | | | b^3 | δ^3 | | | | $\overline{2}$ | $\overline{1}$ | | |
| | δ^3 | | $\overline{1}$ | $\overline{2}$ | | | | | | δ^4 | | | $\overline{2}$ | $\overline{1}$ | | | 1 |
| | δ^4 | | $\overline{1}$ | | | | $\overline{1}$ | | | δ^5 | | $\overline{1}$ | $\overline{1}$ | | 2 | | |
| | δ^5 | | | | | $\overline{1}$ | 1 | | | δ^6 | | | | 1 | | | |

G. F. for invariants, representative form.

$$\text{Denominator : } (1 - b^2)(1 - \delta^2)(1 - \delta^3)(1 - d^2)(1 - d^3)(1 - b^2\delta^2)(1 - b^2\delta) \\ (1 - b^2d^2)(1 - b^2d)(1 - \delta d)(1 - \delta^2d)(1 - d^2\delta).$$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | | | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 |
|-------|------------|-------|----------------|----------------|----------------|----------------|----------------|----------------|--|------------|----------------|----------------|----------------|----------------|----------------|-------|-------|
| | δ^0 | 1 | | | | | | | | δ^3 | | | 1 | | | | |
| | δ^1 | | | | | | | | | δ^4 | | | | | | | |
| b^0 | δ^2 | | | 1 | | | | | | δ^5 | | | | 1 | | | |
| | δ^3 | | | | | | | | | δ^6 | | | | | | | |
| | δ^4 | | | | | 1 | | | | δ^7 | | | | | | | 1 |
| | δ^1 | | 1 | 1 | 1 | | | | | δ^4 | | | 1 | 1 | 1 | | |
| b^1 | δ^2 | | 1 | 1 | 1 | | | | | δ^5 | | | 1 | 1 | 1 | | |
| | δ^3 | | 1 | 1 | 1 | | | | | δ^6 | | | 1 | 1 | 1 | | |
| | δ^1 | | 1 | 1 | | | | | | δ^1 | | $\overline{1}$ | | | | | |
| | δ^2 | | 1 | 1 | | | | | | δ^2 | | $\overline{1}$ | | | | | |
| b^2 | δ^3 | | | | 1 | 1 | | | | δ^3 | $\overline{1}$ | $\overline{1}$ | | 1 | | | |
| | δ^4 | | | | 1 | | $\overline{1}$ | $\overline{1}$ | | δ^4 | | | 1 | 1 | | | |
| | δ^5 | | | | | $\overline{1}$ | | | | δ^5 | | | | | 1 | 1 | |
| | δ^6 | | | | | $\overline{1}$ | | | | δ^6 | | | | | 1 | 1 | |
| | δ^0 | | | | 1 | | | | | δ^2 | | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | |
| | δ^1 | | 1 | 1 | | $\overline{1}$ | $\overline{1}$ | | | δ^3 | | | $\overline{2}$ | $\overline{2}$ | $\overline{1}$ | | |
| b^3 | δ^2 | | 1 | 1 | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | | δ^4 | $\overline{1}$ | $\overline{2}$ | $\overline{3}$ | $\overline{1}$ | | | 1 |
| | δ^3 | 1 | | $\overline{1}$ | $\overline{3}$ | $\overline{2}$ | $\overline{1}$ | | | δ^5 | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | 1 | 1 | | |
| | δ^4 | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | | | | | δ^6 | $\overline{1}$ | $\overline{1}$ | | 1 | 1 | | |
| | δ^5 | | $\overline{1}$ | $\overline{1}$ | $\overline{1}$ | | | | | δ^7 | | | 1 | | | | |

TABLE OF GROUNDFORMS.

| Deg. in coeff's of quadratic. | Deg. in coeff's of quartic. | Deg. in coeff's of quartic. | | | |
|-------------------------------------|-----------------------------------|--------------------------------|---|---|---|
| | | 0 | 1 | 2 | 3 |
| 0 | 0 | | | 1 | 1 |
| | 1 | | 1 | 1 | |
| | 2 | 1 | 1 | 1 | |
| | 3 | 1 | | | |
| 1 | 0 | | | | |
| | 1 | | 1 | 1 | 1 |
| | 2 | | 1 | 1 | 1 |
| | 3 | | 1 | 1 | |
| 2 | 0 | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | |
| | 2 | 1 | 1 | | |
| 3 | 0 | | | | 1 |
| | 1 | | 1 | 1 | |
| | 2 | | 1 | | |
| | 3 | 1 | | | |

SYSTEM OF THREE QUARTICS.

G. F. for invariants, reduced form.

$$\begin{aligned} \text{Denominator: } & (1 - \partial^2)(1 - \partial^3)(1 - \delta^2)(1 - \delta^3)(1 - d^2)(1 - d^3) \\ & (1 - \partial\delta)(1 - \partial d)(1 - \delta d)(1 - \partial^2 d)(1 - \partial d^2) \\ & (1 - \partial^2\delta)(1 - \partial\delta^2)(1 - \delta^2 d)(1 - \delta d^2). \end{aligned}$$

Numerator :

| | | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | | | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 | | |
|--------------|--------------|-------|-------|----------------|----------------|----------------|----------------|----------------|-------|-------|--|--|-------|-------|-------|-------|-------|-------|-------|-------|--|--|
| | ∂^0 | 1 | | | | | | | | | | | | | | | | | | | | |
| | ∂^1 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^2 | | | 1 | | | | | | | | | | | | | | | | | | |
| ∂^0 | ∂^3 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^4 | | | | | 1 | | | | | | | | | | | | | | | | |
| | ∂^1 | | 1 | 1 | | | | | | | | | | | | | | | | | | |
| | ∂^2 | | 1 | 1 | | | | | | | | | | | | | | | | | | |
| | ∂^3 | | | | 1 | 1 | | | | | | | | | | | | | | | | |
| ∂^1 | ∂^4 | | | | 1 | | $\overline{1}$ | $\overline{1}$ | | | | | | | | | | | | | | |
| | ∂^5 | | | | | $\overline{1}$ | | | | | | | | | | | | | | | | |
| | ∂^6 | | | | | $\overline{1}$ | | | | | | | | | | | | | | | | |
| | ∂^0 | | | 1 | | | | | | | | | | | | | | | | | | |
| | ∂^1 | | 1 | 1 | | | | | | | | | | | | | | | | | | |
| | ∂^2 | 1 | 1 | 1 | | | $\overline{1}$ | $\overline{1}$ | | | | | | | | | | | | | | |
| ∂^2 | ∂^3 | | | | 1 | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | | | | | | | | | | | | | |
| | ∂^4 | | | | $\overline{1}$ | $\overline{3}$ | $\overline{2}$ | | 1 | | | | | | | | | | | | | |
| | ∂^5 | | | $\overline{1}$ | $\overline{2}$ | $\overline{2}$ | 1 | 1 | | | | | | | | | | | | | | |
| | ∂^6 | | | $\overline{1}$ | $\overline{1}$ | | 1 | 1 | | | | | | | | | | | | | | |
| | ∂^7 | | | | | 1 | | | | | | | | | | | | | | | | |
| | ∂^1 | | | | 1 | 1 | | | | | | | | | | | | | | | | |
| | ∂^2 | | | | 1 | $\overline{1}$ | $\overline{2}$ | $\overline{1}$ | | | | | | | | | | | | | | |
| ∂^3 | ∂^3 | | 1 | 1 | | $\overline{4}$ | $\overline{3}$ | $\overline{1}$ | | | | | | | | | | | | | | |
| | ∂^4 | | 1 | $\overline{1}$ | $\overline{4}$ | $\overline{5}$ | | 2 | 1 | | | | | | | | | | | | | |
| | ∂^5 | | | $\overline{2}$ | $\overline{3}$ | | 3 | 2 | | | | | | | | | | | | | | |
| | ∂^6 | | | $\overline{1}$ | $\overline{1}$ | 2 | 2 | 1 | | | | | | | | | | | | | | |
| | ∂^7 | | | | | 1 | | | | | | | | | | | | | | | | |
| | ∂^4 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^5 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^6 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^7 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^1 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^2 | | | | | | | | | | | | | | | | | | | | | |
| ∂^5 | ∂^3 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^4 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^5 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^6 | | | | | | | | | | | | | | | | | | | | | |
| | ∂^7 | | | | | | | | | | | | | | | | | | | | | |

Numerator—Continued :

| | d^0 | d^1 | d^2 | d^3 | d^4 | d^5 | d^6 | d^7 | d^8 |
|------------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| δ^0 | | | | | 1 | | | | |
| δ^1 | | | | 1 | | $\overline{1}$ | $\overline{1}$ | | |
| δ^2 | | | | $\overline{1}$ | $\overline{3}$ | $\overline{2}$ | | 1 | |
| δ^3 | | 1 | $\overline{1}$ | $\overline{4}$ | $\overline{5}$ | | 2 | 1 | |
| δ^4 | 1 | | $\overline{3}$ | $\overline{5}$ | | 5 | 3 | | $\overline{1}$ |
| δ^5 | | $\overline{1}$ | $\overline{2}$ | | 5 | 4 | 1 | $\overline{1}$ | |
| δ^6 | | $\overline{1}$ | | 2 | 3 | 1 | | | |
| δ^7 | | | 1 | 1 | | $\overline{1}$ | | | |
| δ^8 | | | | | $\overline{1}$ | | | | |

Representative form same as reduced form.

TABLE OF GROUNDFORMS.

| Deg. in coeff's of quartic | Deg. in coeff's of quartic | Deg. in coeff's of quartic. | | | |
|----------------------------------|----------------------------------|--------------------------------|---|---|---|
| | | 0 | 1 | 2 | 3 |
| 0 | 0 | | | 1 | 1 |
| | 1 | | 1 | 1 | |
| | 2 | 1 | 1 | 1 | |
| | 3 | 1 | | | |
| 1 | 0 | | 1 | 1 | |
| | 1 | 1 | 1 | 1 | |
| | 2 | 1 | 1 | 1 | |
| 2 | 0 | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | |
| | 2 | 1 | 1 | | |
| 3 | 0 | 1 | | | |