69.

ON A CERTAIN INTEGRABLE CLASS OF DIFFERENTIAL AND FINITE DIFFERENCE EQUATIONS.

[*Johns Hopkins University Circulars*, I. (1882), p. 178.]

IN Mr Moulton's edition of Boole's *Finite Differences* will be found quoted from the author of this notice a certain class of equations of which the *general* integral can be found as for example

$$\begin{vmatrix} u_x & u_{x+1} \\ u_{x+1} & u_{x+2} \end{vmatrix} = A\alpha^x,$$

that is,
$$u_x u_{x+2} - u^2{}_{x+1} = A\alpha^x,$$

or again
$$\begin{vmatrix} u_x & u_{x+1} & u_{x+2} \\ u_{x+1} & u_{x+2} & u_{x+3} \\ u_{x+2} & u_{x+3} & u_{x+4} \end{vmatrix} = A\alpha^x,$$

and so on for a persymmetrical determinant of any order (n) constructed on the same principle as the two foregoing ones; an equation of the nth degree and $2n$th order will thus arise.

In this communication to the Seminarium the writer pointed out that an integral (but without any arbitrary constants) may be found for an equation of the same form as that above indicated on the left hand side but with $(n+1)$ different exponentials instead of a single one on the right hand side as for example

$$u_x u_{x+2} - u^2{}_{x+1} = A\alpha^x + B\beta^x + C\gamma^x$$

can be integrated provided that there are really three and not merely two distinct terms as would happen if A or B or C were one of them to vanish. But any number of the exponentials may be made indefinitely near to each other and the integral still hold good; in this way other integrable forms of equations can be obtained. As for instance

$$u_x u_{x+2} - u^2{}_{x+1} = A\alpha^x + (B + Cx)\,\beta^x,$$
$$u_x u_{x+2} - u^2{}_{x+1} = (A + Bx + Cx^2)\,\alpha^x$$

are integrable.

The same conclusions in all respects apply both as regards the general and the special integral case when any term u_x is replaced by y and u_{x+i} by $\left(\dfrac{d}{dx}\right)^i y$. The form of the special integral whether for differential or difference equations is rather too long to produce in this abstract but will be given in full in a future number of the *American Journal of Mathematics* [above, p. 546].