72.

ON THE GEOMETRICAL FORMS CALLED TREES.

[*Johns Hopkins University Circulars*, I. (1882), pp. 202, 203.]

[IN connexion with the reference to his name in the above] Professor Sylvester stated that to M. Camille Jordan was due the credit of being the first to discover the existence of the centre or centre-pair of each kind described in the above note. In entire ignorance of M. Jordan's work he rediscovered for himself the centre or centre-pair of the first kind, and was the first to make use of the method immediately flowing therefrom to solve the problem of finding the forms and the number of tree-graphs* corresponding to a hydro-carbon or hypothetical hydro-boron series with a given number of carbon atoms. His results, which he communicated from time to time to Professor Tait, of Edinburgh, were however as regards the ascertainment of the number of such graphs, purely arithmetical, but giving all the different forms of the so-called trees or (more properly speaking) ramifications for different values of the number of atoms up to a certain arithmetical limit. The problem was subsequently taken up from this point by Professor Cayley, who obtained general generating-function formulæ for effecting the denumeration of the graphs. Mr Sylvester then proceeded to explain his method of arriving at the first kind of centre or centre-pair of any given tree or ramification.

To this end he supposes all the terminal branches of the tree removed. A tree with a less number of nodes is thus brought into evidence which is subjected (if possible) to like treatment and so a third tree with still fewer nodes is arrived at. As this process cannot be indefinitely continued (for if so a finite number could be continually diminished) we must at length come

* In accordance with the nomenclature employed above, the writer uses here occasionally the word *tree*, but considers his original word *ramification* more correct. A tree is a ramification with one point fixed as a root or origin, and no such fixed origin is supposed to exist in the graphs in question.

to a tree or ramification whose terminal branches cannot be removed without leaving nothing in the form of a tree remaining. So long as not less than three nodes remain, since they must not form a triangle, for that would be inconsistent with their appertaining to a ramification, the process of lopping off terminals cannot be brought to a close. Eventually, therefore, this process must lead to a system of branches all radiating out from a single point, or which being removed, only an isolated point remains, or else to a sort of double-headed mop or broom consisting of two such radiating systems stuck into the two ends of an axis. This is the case of bicentric or axial, the former of a monocentric ramification. Thus every ramification may be said to belong either to a central or an axial class. He concluded with suggesting that some general chemical or physical property or set of such properties might reasonably be supposed to exist serving to distinguish between these two classes or genera in the case of the well developed series of the hydrocarbons.