73.

ON THE 8-SQUARE IMAGINARIES.

[*Johns Hopkins University Circulars*, I. (1882), p. 203.]

[WITH reference to the above communication] Professor Sylvester referred to the general question of representing the product of sums of two, four or eight squares under the form of a like sum, and mentioned that Professor Cayley had been the first to demonstrate, by an exhaustive investigation, the impossibility of extending the law applicable to 2, 4 and 8 to the case of 16 squares. The new kind of so-called imaginaries referred to by Professor Cayley are, as far as Mr Sylvester is aware, the first example of the introduction into Analysis of locative symbols not subject to the strict law of association, and he considers the law regulating the connexion of the two products represented by a succession of three such symbols, most interesting, inasmuch as such products are either identical, or if not identical, of the same absolute value, but with contrary signs: most persons, before this example had been brought forward, would have felt inclined to doubt the possibility of locative symbols (*vulgo* imaginary quantities*) whose multiplication table should give results inconsistent with the common associative

* Using θ, h, t, u to denote thousands, hundreds, tens, units, the year of grace in which we live may be represented by $\theta+8h+8t+2u$, θ, h, t, u, being locative symbols which it would be absurd to style *imaginary quantities*; but they are as much entitled to that name as the i, j, k, or any like set of symbols—the only essential difference being that the one set of symbols is limited, the other unlimited in number—and accordingly the law of combination of the one set is given by a finite and of the other set by an infinite *multiplication table*. We might mark off the specific difference between the two cases, by defining the latter set as *unlimited*, the former as *recurrent* or *periodic* locatives or locators; the *locatives* indicate out of what *basket*, so to say, the *quantities* appearing in an analytical expression are to be selected—the multiplication table determines the basket into which their product is to be thrown. Under a purely analytical point of view this is all that is wanted—but in the application of quaternions to problems in nature, it becomes necessary to give special significance to the baskets or rubrics (which would do as well) to which the quantities belong and understand them to signify that certain geometrical processes of *setting* are to be performed.

The true analytical theory of quaternions has nothing to do with this setting part of the law, being capable of forming the groundwork of any real accession to algebraical science—the results of Professor Cayley referred to above, seem to show that such doubts are open to question. Mr Sylvester mentioned as bearing upon the subject of so-called imaginary quantities, that in his recent researches in Multiple Algebra he had come upon a system of Nonions, the exact analogues of the Hamiltonian Quaternions and like them capable of being represented by square matrices. Mr Charles S. Peirce, it should be stated, had to the certain knowledge of Mr Sylvester arrived at the same result many years ago in connexion with his theory of the *logic of relatives*; but whether this result had been published by Mr Peirce, he was unable to say*.

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business, and regards quaternions as matrices of the second order of a certain determinate form, and accordingly the whole analytical side of the theory of quaternions merges into a particular case of the general theory of *Multiple Algebra*.

As far as the present writer is aware, Professor Cayley in his memoir on Matrices, (*Phil. Trans.* 1858), was the first to recognize the parallelism between quaternions and matrices, but the idea and method of effecting their complete identification is due to the late Prof. Benjamin Peirce or to his son Mr C. S. Peirce.

* Mr C. S. Peirce gave a form of this Algebra in a paper "On a Notation for the Logic of Relatives," published in 1870. The class of Associative Algebras to which this belongs were termed *quadrates* by the late Professor Clifford.