# 87.

## NOTE ON THE GRAPHICAL METHOD IN PARTITIONS.

[*Johns Hopkins University Circulars*, II. (1883), pp. 70, 71.]

It is well I think to draw attention to the fact that the graphical method introduces two new processes into Arithmetic as elementary and fundamental as those contained in the well-known four rules—which may be called Transversion and Apocopation.

*Transversion* is the operation of passing from a partition to its conjugate or transverse, and is identical with that which borrowing from the vernacular of the American Stock Exchange I have elsewhere denominated "calling."

The elements of a partition may be regarded as *Sellers* each holding a certain number of shares in the same stock. On the numbers 1, 2, 3 ... being successively called out each seller who holds at least that number of shares declares himself, and the number of those so responding each time being set down, a new partition is formed with numbers whose sum is identical with the total number of shares on sale.

The discovery of this process is due to Dr Ferrers, who informs me that he himself never published it but left it to me to do so in his name in the *London and Edinburgh Philosophical Magazine* for 1853*. I may mention that I have never missed an opportunity of expressing my sense of the great importance of the discovery and bringing it under the notice of my pupils, to one of whom, Mr Durfee (Fellow of this University), is due the discovery (after the lapse of 30 years) which leads to the second process, namely, Apocopation, which institutes a fixed relation between any partition and its transverse.

Apocopation is a process applied to a partition whose parts are arranged in descending order and consists in cutting off from its beginning, all those terms whose magnitude exceeds the number which denotes their place (reckoning from the highest term) in the arrangement. We have then

[* Vol. I. of this Reprint, p. 597; Vol. II., p. 120.]

this important theorem—*The number of terms subject to apocopation is the same for any partition and its transverse.*

*Scaling* or *co-summating* a partition consists in adding together each-to-each the apocopated terms in a partition and in its transverse, and diminishing these sums by the several numbers 1, 3, 5 .... In this way a new partition is formed which may be called the associate-sum of the original partition, so that to every partition there is a transverse and an associate-sum; and the content of each of these three partitions is identical.

The process of *scaling* or co-summation may be indefinitely continued and it is a curious question to determine how often the scaling process must be continued in order for a given partition to be eventually converted into a single term after which of course it remains unaltered by any further application of the process—this problem is naturally suggested by the practice of scaling and rescaling an inconveniently large public debt which is sometimes practised in the Old World and is not unknown in the New; but the analogy fails in this respect that in the one case the amount of the debt has a tendency to converge to zero, whereas in the other the content of the partition remains constant throughout.

The passage from a partition into odd numbers to the corresponding partition into unequal numbers, is effected by a co-summation operated simultaneously but independently upon two partitions, one of which has for its parts the major-halves and the other the minor-halves of the parts of the given partition.