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NOTE ON THE HOMOLOGY OF SETS.

[From the *Quarterly Mathematical Journal*, vol. I. (1857), p. 178.]

LET L denote a set of any four elements a, b, c, d, and in like manner Λ, L_1 &c. sets of the four elements $\alpha, \beta, \gamma, \delta$; $a_{,}, b_{,}, c_{,}, d_{,}$, &c.; then we may establish a relation of homology between four sets L, L_1, L_2, L_3, and four other sets $\Lambda, \Lambda_1, \Lambda_2, \Lambda_3$; viz., considering the corresponding anharmonic ratios of the different sets, we may suppose a relation of homology between these ratios. Thus considering the set to L, write

$$x=(a-b)(c-d),$$
$$y=(a-c)(d-b),$$
$$z=(a-d)(b-c),$$

then $x+y+z=0$ and the anharmonic ratios of the set are $x:y:z$—we may, if we please, take $x:y$ as the anharmonic ratio of the set. And in like manner taking $\xi:\eta$ as the anharmonic ratio of the set $\alpha, \beta, \gamma, \delta$, &c., the assumed relation between the sets L, L_1, L_2, L_3 and the sets $\Lambda, \Lambda_1, \Lambda_2, \Lambda_3$ will be

$$\begin{vmatrix} x\xi, & x\eta, & y\xi, & y\eta \\ x_1\xi_1, & x_1\eta_1, & y_1\xi_1, & y_1\eta_1 \\ x_2\xi_2, & x_2\eta_2, & y_2\xi_2, & y_2\eta_2 \\ x_3\xi_3, & x_3\eta_3, & y_3\xi_3, & y_3\eta_3 \end{vmatrix}=0;$$

and it is to be observed, that this relation is independent of the particular ratio $x:y$ which has been chosen as the anharmonic ratio of the set; in fact, if we write $x=-y-z$, $\xi=-\eta-\zeta$, &c., then reducing the result by means of an elementary property of determinants, the equation will preserve its original form, but will contain the ratios $y:z$; $\eta:\zeta$, &c., instead of the ratios $x:y$; $\xi:\eta$, &c.