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SOLUTION OF A MECHANICAL PROBLEM.

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A HEAVY plane is supported by parallel elastic strings of small extensibility; and the strings are of the same length and extensibility: required the position of equilibrium.

Imagine the plane horizontal, and let n be the number of strings, (a, b), (a', b'), &c. the coordinates of the points of attachment; ξ, η the coordinates of the centre of gravity of the plane; W the weight; let the equation of the horizontal line about which the plane turns be

$$x \cos \alpha + y \sin \alpha - p = 0;$$

and let $\delta\theta$ be the inclination of the plane in its position of equilibrium to the horizontal plane, and $\omega\delta l$ the force generated by an increase δl in the length of one of the strings.

We have for the conditions of equilibrium

$$\Sigma (a \cos \alpha + b \sin \alpha - p) \ \omega\delta\theta - W \ = 0,$$
$$\Sigma (a \cos \alpha + b \sin \alpha - p) \ a\omega\delta\theta - W\xi = 0,$$
$$\Sigma (a \cos \alpha + b \sin \alpha - p) \ b\omega\delta\theta - W\eta = 0;$$

or putting $\Sigma a = L$, $\Sigma b = M$, $\Sigma a^2 = A$, $\Sigma ab = H$, $\Sigma b^2 = B$, we have

$$L \cos \alpha + M \sin \alpha - np - \frac{W}{\omega\delta\theta} = 0,$$

$$A \cos \alpha + H \sin \alpha - Lp - \frac{W\xi}{\omega\delta\theta} = 0,$$

$$H \cos \alpha + B \sin \alpha - Mp - \frac{W\eta}{\omega\delta\theta} = 0.$$

Combining with these the equations

$$x \cos \alpha + \eta \sin \alpha - p = 0,$$

and eliminating linearly $\cos \alpha$, $\sin \alpha$, p and W, we have

$$\begin{vmatrix} \cdot & x, & y, & 1 \\ \xi, & A, & H, & L \\ \eta, & H, & B, & M \\ 1, & L, & M, & n \end{vmatrix} = 0$$

for the equation of the required line $x \cos \alpha + y \sin \alpha - p = 0$. Replacing L, M, A, H, B by their values, the equation is readily transformed into

$$\Sigma \left\{ \begin{vmatrix} x, & y, & 1 \\ a, & b, & 1 \\ a', & b', & 1 \end{vmatrix} \times \begin{vmatrix} \xi, & \eta, & 1 \\ a, & b, & 1 \\ a', & b', & 1 \end{vmatrix} \right\} = 0$$

where the summation extends to each pair of points (a, b) and (a', b'). This is, in fact, an extension of the harmonic relation of a point and line with respect to a triangle.