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NOTE ON THE THEORY OF ATTRACTION.

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IMAGINE a closed surface, the equation of which contains the two parameters m, h. Call this the surface (m, h), and suppose also that for shortness the shell of uniform density included between the surfaces (m, h), (n, h) is called the shell (m, n, h). Suppose now that the surface is such:

1°. That the infinitesimal shell $(m, m+dm, h)$ exerts no attraction upon an internal point.

2°. That the equipotential surfaces of the shell in question for external points are the surfaces (m, k), where k is arbitrary.

Then, first, the attraction of the shell on a point of the equipotential surface (m, k) is proportional to the normal thickness at that point of the shell $(m, m+\delta m, k)$; or (more precisely) taking the density of the attracting shell as unity, the attraction is $= 4\pi \times$ mass of shell $(m, m+dm, h)$ into normal thickness of shell $(m, m+\delta m, k)$ divided by mass of the last-mentioned shell.

In fact the shell $(m, m+\delta m, k)$ exerts no attraction on an internal point, consequently if over the surface (m, k) we distribute the mass of the original shell $(m, m+dm, h)$ in such manner that the density at any point is proportional to the normal thickness of the shell $(m, m+\delta m, k)$ the distribution will be such that the attraction on an internal point may vanish; but in order that this may be the case, the density must be equal to $\frac{1}{4\pi}$ into the attraction upon that point of the shell $(m, m+\delta m, k)$. Hence the attraction is proportional to the normal thickness, and if the whole mass distributed over the surface (m, k) is precisely equal to the mass of the shell $(m, m+dm, h)$, then the density at any point must be equal to the mass into normal thickness divided by mass of $(m, m+\delta m, k)$, and attraction $=4\pi$ into density, $=4\pi\times$ mass of shell $(m, m+dm, h)$ into normal thickness of shell $(m, m+\delta m, k)$ divided by mass of the last-mentioned shell.

And, secondly, the attractions of the solids bounded by the two surfaces (n, h), (n, h_1) respectively upon the same exterior point are proportional to their masses.

For the solid (n, h) may be divided into shells $(m, m+dm, h)$ and for this shell the equipotential surface is (m, k) and the attraction of the shell varies as mass of $(m, m+dm, h)$ into normal thickness of the shell $(m, m+\delta m, k)$. But in like manner the solid (n, h_1) may be divided into shells $(m, m+dm, h_1)$ and the attraction of the shell varies as mass of $(m, m+dm, h_1)$ into normal thickness of the shell $(m, m+\delta m, k)$ and the attractions are in each case in the direction normal to the shell (m, k), and therefore in the same direction; that is, the attraction of the shell $(m, m+dm, h)$ is in the same direction as that of the shell $(m, m+dm, h_1)$ and the two attractions are proportional to the masses. Hence integrating from $m=0$ (if for this value the included space is zero) to $m=n$, the attractions of the solids (n, h), (n, h_1) are composed of elements proportional and parallel, the elements of the attraction of (n, h) to the elements of the attraction of (n, h_1); and consequently the total attractions are in the same direction and proportional to the masses.

Thirdly, the attractions of the two surfaces (m, h), (n, h) upon the same interior point are equal.

A surface having the properties in question is of course the ellipsoidal surface

$$\frac{x^2}{m(a^2+h)}+\frac{y^2}{m(b^2+h)}+\frac{z^2}{m(c^2+h)}=1,$$

where if m varies (h being constant) the several surfaces are similar to each other, but if h varies (m being constant) the several surfaces are confocal to each other: for it is in fact well known that the infinitesimal shell bounded by similar ellipsoidal surfaces has the properties assumed with respect to the shell $(m, m+dm, h)$. The first theorem in effect reduces the problem of the determination of an ellipsoid upon an exterior point to a single integration, and constitutes the foundation of Poisson's method for the attraction of ellipsoids. The second theorem (Maclaurin's theorem for the attraction of ellipsoids on the same external point) shows that the attraction of an ellipsoid upon an external point can be found by means of the attraction of the confocal ellipsoid through the attracted point; and by the third theorem the attraction of an ellipsoid upon an interior point is equal to that of the similar ellipsoid through the attracted point; hence the second and third theorems reduce the determination of the attraction of an ellipsoid upon an external or internal point to that of an ellipsoid upon a point on the surface.

2, *Stone Buildings*, *W.C.*, *7th April*, 1858.