198.

NOTE UPON A RESULT OF ELIMINATION.

[From the *Philosophical Magazine*, vol. XI. (1856), pp. 378—379.]

If the quadratic function

$$(a, b, c, f, g, h\between x, y, z)^2$$

break up into factors, then representing one of these factors by $\xi x+\eta y+\zeta z$, and taking any arbitrary quantities α, β, γ, the factor in question, and therefore the quadratic function is reduced to zero by substituting $\beta\zeta-\alpha\eta$, $\gamma\xi-\alpha\zeta$, $\alpha\eta-\beta\zeta$ in the place of x, y, z. Write

$$(a, b, c, f, g, h\between \beta\zeta-\alpha\eta,\ \gamma\xi-\alpha\zeta,\ \alpha\eta-\beta\zeta)^2=(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{f}, \mathrm{g}, \mathrm{h}\between \alpha, \beta, \gamma)^2;$$

the coefficients of the function on the right hand are

$$\begin{array}{lllllll}
\mathrm{a}= & . & c\eta^2 & +b\zeta^2 & -2f\eta\zeta & . & , \\
\mathrm{b}= & c\xi^2+ & . & +a\zeta^2 & . & -2g\zeta\xi & , \\
\mathrm{c}= & b\xi^2+a\eta^2 & . & . & . & -2h\xi\eta, & \\
\mathrm{f}= & -f\xi^2 & . & . & -a\eta\zeta+ & h\zeta\xi+ & g\xi\eta, \\
\mathrm{g}= & . & g\eta^2 & . & +h\eta\zeta- & b\zeta\xi+ & f\xi\eta, \\
\mathrm{h}= & . & . & -h\zeta^2 & +g\eta\zeta+ & f\zeta\xi- & c\xi\eta;
\end{array}$$

and it is to be remarked that we have identically

$$\mathrm{a}\xi+\mathrm{h}\eta+\mathrm{g}\zeta=0,$$

$$\mathrm{h}\xi+\mathrm{b}\eta+\mathrm{f}\zeta=0,$$

$$\mathrm{g}\xi+\mathrm{f}\eta+\mathrm{c}\zeta=0.$$

Hence of the six equations, $\mathrm{a}=0$, $\mathrm{b}=0$, $\mathrm{c}=0$, $\mathrm{f}=0$, $\mathrm{g}=0$, $\mathrm{h}=0$, any three (except $\mathrm{a}=0$, $\mathrm{h}=0$, $\mathrm{g}=0$, or $\mathrm{h}=0$, $\mathrm{b}=0$, $\mathrm{f}=0$, or $\mathrm{g}=0$, $\mathrm{f}=0$, $\mathrm{c}=0$) imply the remaining three.

If from the six equations we eliminate ξ^2, η^2, &c., we obtain

$$\square = \begin{vmatrix} . & c, & b, & -2f, & . & . \\ c, & . & a, & . & -2g, & . \\ b, & a, & . & . & . & -2h \\ -f, & . & . & -a, & h, & g \\ . & -g, & . & f, & -b, & f \\ . & . & -h, & g, & c, & -c \end{vmatrix} = 0;$$

and the equation $\square = 0$ is therefore the result of the elimination of ξ, η, ζ from any three (other than the excepted combinations) of the six equations. But from what precedes, it appears that the equation $\square = 0$ must be satisfied when the quadratic function breaks up into factors, and consequently $\square$ must contain as a factor the discriminant

$$K = \begin{vmatrix} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{vmatrix}$$

of the quadratic function. This agrees perfectly with the results obtained long ago by Prof. Sylvester in his paper, "Examples of the Dialytic Method of Elimination as applied to Ternary Systems of Equations," *Camb. Math. Journ.* vol. II. p. 232; but according to the assumption there made, the value of $\square$ would be (to a numerical factor *près*) $= abcK$. The correct value is by actual development shown to be $\square = -2K^2$. It would be interesting to show *à priori* that $\square$ contains K^2 as a factor.

2, *Stone Buildings, March* 28, 1856.