XXVI

ON AN EQUATION OF THE ELLIPSOID

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The Secretary of Council read the following communication from Sir William Rowan Hamilton, on an equation of the ellipsoid.

'A remark of yours, recently made, respecting the form in which I first gave to the Academy, in December 1845, an equation* of the ellipsoid by quaternions—namely, that this form involved only *one* asymptote of the focal hyperbola—has induced me to examine, simplify, and extend, since I last saw you, some manuscript results of mine on that subject; and the following new form of the equation, which seems to meet your requisitions, may, perhaps, be shewn to the Academy tonight. This new form is the following:

$$\mathrm{TV}\frac{\eta\rho-\rho\theta}{\mathrm{U}(\eta-\theta)}=\theta^2-\eta^2. \tag{1}$$

'The constant vectors η and θ are in the directions of the two asymptotes required; their symbolic sum, $\eta+\theta$, is the vector of an umbilic; their difference, $\eta-\theta$, has the direction of a cyclic normal; another umbilicar vector being in the direction of the sum of their reciprocals, $\eta^{-1}+\theta^{-1}$, and another cyclic normal in the direction of the difference of those reciprocals, $\eta^{-1}-\theta^{-1}$. The lengths of the semiaxes of the ellipsoid are expressed as follows:

$$a=\mathrm{T}\eta+\mathrm{T}\theta;\quad b=T(\eta-\theta);\quad c=T\eta-\mathrm{T}\theta. \tag{2}$$

'The focal ellipse is given by the system of the two equations

$$\mathrm{S}.\rho\mathrm{U}\eta=\mathrm{S}.\rho\mathrm{U}\theta; \tag{3}$$

and

$$\mathrm{TV}.\rho\mathrm{U}\eta=2\mathrm{S}\sqrt{(\eta\theta)}; \tag{4}$$

where $\mathrm{TV}.\rho\mathrm{U}\eta$ may be changed to $\mathrm{TV}.\rho\mathrm{U}\theta$; and which represent respectively a plane, and a cylinder of revolution. Finally, I shall just add what seems to me remarkable—though I have met with several similar results in my unpublished researches—that the focal hyperbola is adequately represented by the *single* equation following:

$$\mathrm{V}.\eta\rho.\mathrm{V}.\rho\theta=(\mathrm{V}.\eta\theta)^2.\text{'} \tag{5}$$

* [See XLII.]