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QUATERNION PROOF OF A THEOREM OF RECIPROCITY OF CURVES IN SPACE*

[*British Assoc. Report*, pt. II (1862), p. 4.]

Let ϕ and ψ be any two vector functions of a scalar variable, and ϕ', ψ', ϕ'', ψ'' their derived functions, of the first and second orders. Then each of the two systems of equations, in which c is a scalar constant,

$$(1)\quad S\phi\psi=c,\quad S\phi'\psi=0,\quad S\phi''\psi=0,$$

$$(2)\quad S\psi\phi=c,\quad S\psi'\phi=0,\quad S\psi''\phi=0,$$

or each of the vector expressions,

$$(3)\quad \psi=\frac{cV\phi'\phi''}{S\phi\phi'\phi''},\qquad (4)\quad \phi=\frac{cV\psi'\psi''}{S\psi\psi'\psi''},$$

includes the other.

If then, from any assumed origin, there be drawn lines to represent the reciprocals of the perpendiculars from that point on the osculating planes to a first curve of double curvature, those lines will terminate on a second curve, from which we can return to the first by a precisely similar process of construction.

And instead of thus taking the reciprocal of a curve with respect to a sphere, we may take it with respect to any surface of the second order, and is probably well known to geometers, although the author was lately led to perceive it for himself by the very simple analysis given above.

* [See *Elements*, article 385.]