XLVI

ON A GENERAL CENTRE OF APPLIED FORCES*

Communicated 22 June 1863.

[*Proc. Roy. Irish Acad.* vol. VIII (1864), p. 394.]

Sir W. R. Hamilton wishes a note to be preserved in the *Proceedings* of the Royal Irish Academy, that on recently reconsidering an application of Quaternions to the Statics of a Solid Body, some account of which was laid before the Academy many years ago (see the *Proceedings*† for December 1845), he has been led to perceive the *theoretical* (and to suspect the *practical*) existence of a certain *Central Point* for *every system of applied forces*, not reducible to a *couple*, nor to *zero*: which *generally new point*, for the case of *parallel forces*, coincides with their well-known *centre*.

An *applied force AB*, acting at a point A, being said to have a *quaternion moment*, equal to the quaternion *product OA . AB*, with respect to any assumed point O, the *sum* of all *such* moments, or the quaternion, $Q = \Sigma(OA . AB) = OA . AB + OA' . A'B' + \&c.$, is called the *total quaternion moment* of the applied system with respect to the same point O.

This *total moment* Q *varies* generally with the *point* to which it is referred; and there is *one* point C, or *one position* of O, for which the condition

$$TQ = a \ minimum,$$

is satisfied, with the exceptions (of *couple* and *equilibrium*) above alluded to.

It is *this point C*, which Sir W. R. H. proposes to call *generally* the *Centre of a System of Applied Forces*.

In the most general case of such a system, he finds it to be situated *on the Central Axis*, the *minimum* TQ representing then what was called by Poinsot the *Energy of the Central Couple*.

For the less general case of an *unique resultant force*, the quaternion Q reduces itself to *zero* at the new Central Point C, which is now situated *on the resultant*, and determines its *line of application*.

* [See XLII, pp. 452, 453.] † [See *Elements*, article 416.]