LVI

MEMORANDUM FOR JOHN T. GRAVES, ESQ. FROM HIS FRIEND THE AUTHOR

[Note-book 139.]

It has lately occurred to me, that several investigations respecting polyhedra may be assisted by the introduction of a new system of symbols, ι, κ, λ, which shall be regarded as roots of positive unity, but being non-commutative, although associative, as factors among themselves. For example, I find that if we assume

$$\iota^2 = 1, \quad \kappa^3 = 1, \quad \lambda^5 = 1, \quad \lambda = \iota\kappa \tag{A}$$

then the only cyclic mode of passing over all the successive faces of the Icosahedron, or (along twenty of the thirty edges) from corner to corner of the Dodecahedron, is fully represented by the formula,

$$(k_1^3 k_2^3 (k_1 k_2)^2)^2 = 1; \tag{B}$$

where

$$k_1 = \iota\kappa, \quad k_2 = \iota\kappa^2. \tag{C}$$

By making similarly

$$l_1 = \iota\lambda, \quad l_2 = \iota\lambda^2, \quad l_3 = \iota\lambda^3, \quad l_4 = \iota\lambda^4 \tag{D}$$

so that

$$k_1 = \lambda, \quad \text{and} \quad l_1 = \kappa \tag{E}$$

while k_2 and $l_2 l_3 l_4$ have the significations stated above, I find that all results respecting passages over successive faces of the dodecahedron, or along successive edges of the icosahedron, so as to cover all, or some, of the faces of the one solid, or to pass over all, or some, of the corners of the other, admit of being expressed by formulae of the same general kind as (B), with l's instead of k's for factors. For other solids I use other exponents.

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7 *October* 1856.