IV.

§ 1.

Def.

(§ 2) 1. $n \varepsilon \mathrm{N} . f \varepsilon \mathrm{G}/\mathrm{Z}_{n+1} . \supset . \Sigma_1^{n+1} f = \Sigma_1^{n} f + f(n+1)$

(§ 3) 1. $\mathrm{A}, \mathrm{B} \varepsilon \mathrm{G} . \supset : \mathrm{A} > \mathrm{B} . = . \mathrm{A} \varepsilon \mathrm{B} + \mathrm{G}$

2. » $. \supset : \mathrm{A} < \mathrm{B} . = . \mathrm{B} > \mathrm{A}$

(§ 4) $\mathrm{A}, \mathrm{B}, \mathrm{C} \varepsilon \mathrm{G} . \supset :$

1. $\mathrm{A} - \mathrm{B} = \mathrm{G} \frown \overline{\mathrm{X}\, \varepsilon} (\mathrm{A} = \mathrm{B} + \mathrm{X})$

2. $\mathrm{A} - \mathrm{B} + \mathrm{C} = (\mathrm{A} - \mathrm{B}) + \mathrm{C}$

3. $\mathrm{A} + \mathrm{B} - \mathrm{C} = (\mathrm{A} + \mathrm{B}) - \mathrm{C}$

4. $\mathrm{A} - \mathrm{B} - \mathrm{C} = (\mathrm{A} - \mathrm{B}) - \mathrm{C}$

(§ 5) 1. $\mathrm{A} \varepsilon \mathrm{G} . \supset . 1\mathrm{A} = \mathrm{A}$

2. » $n \varepsilon \mathrm{N} . \supset . (n+1)\mathrm{A} = n\mathrm{A} + \mathrm{A}$

(§ 6) 1. $\mathrm{A}, \mathrm{B} \varepsilon \mathrm{G} . m, n \varepsilon \mathrm{N} . \supset : \mathrm{A} = \frac{m}{n} \mathrm{B} . = . n\mathrm{A} = m\mathrm{B}$

2. $\mathrm{A} \varepsilon \mathrm{G} . a \varepsilon \mathrm{R} . m, n \varepsilon \mathrm{N} . \mathrm{D}(m, n) = 1 . a = \frac{m}{n} . \supset . a\mathrm{A} = \frac{m}{n} \mathrm{A}$

(§ 7) 1. $\mathrm{A} \varepsilon \mathrm{KG} . \mathrm{X} \varepsilon \mathrm{G} . \supset : \mathrm{X} < \mathrm{l}'\mathrm{A} . = . \mathrm{A} \frown (\mathrm{X} + \mathrm{G}) -= \Lambda$

2. » $\mathrm{B} \varepsilon \mathrm{G} . \supset :: \mathrm{l}'\mathrm{A} = \mathrm{B} . = .\!\cdot . \mathrm{X} \varepsilon \mathrm{G} . \supset_{\mathrm{X}} : \mathrm{X} < \mathrm{l}'\mathrm{A} . = . \mathrm{X} < \mathrm{B}$

(§ 8) 1. $\mathrm{A} \varepsilon \mathrm{G} . a \varepsilon \mathrm{KR} . \mathrm{l}'a \varepsilon \mathrm{Q} . \supset . (\mathrm{l}'a)\mathrm{A} = \mathrm{l}'(a\mathrm{A})$

2. » $m \varepsilon \mathrm{Q} . \supset : m\mathrm{A} = \{ \mathrm{l}'(\mathrm{R} \frown \overline{x\, \varepsilon} (x \stackrel{=}{<} m)) \} \mathrm{A}$

(§ 9) 1. $\mathrm{A}, \mathrm{U} \varepsilon \mathrm{G} . \supset . \mathrm{A}/\mathrm{U} = \mathrm{l}' \{ \mathrm{R} \frown \overline{x\, \varepsilon} (n, nx \varepsilon \mathrm{N} . (nx)\mathrm{U} \stackrel{=}{<} n\mathrm{A} . -=_n \Lambda) \}$

2. » $. \supset . \bar{\mathrm{l}}'(\mathrm{A}/\mathrm{U}) = \mathrm{R} \frown \overline{x\, \varepsilon} (n, nx \varepsilon \mathrm{N} . (nx)\mathrm{U} \stackrel{=}{<} n\mathrm{A} . -=_n \Lambda)$

Pp.

$\mathrm{A, B, C} \,\varepsilon\, \mathrm{G} \,.\, \supset :$

0. $\mathrm{A} + \mathrm{B} \,\varepsilon\, \mathrm{G}$

1. $\mathrm{A} = \mathrm{B} \,.\, \supset .\, \mathrm{A} + \mathrm{C} = \mathrm{B} + \mathrm{C}$

1_1. $\text{»} \quad .\, \supset .\, \mathrm{C} + \mathrm{A} = \mathrm{C} + \mathrm{B}$

2. $\mathrm{A} + \mathrm{C} = \mathrm{B} + \mathrm{C} \,.\, \supset .\, \mathrm{A} = \mathrm{B}$

2_1. $\mathrm{C} + \mathrm{A} = \mathrm{C} + \mathrm{B} \,.\, \supset .\, \mathrm{A} = \mathrm{B}$

3. $\mathrm{A} + \mathrm{B} = \mathrm{B} + \mathrm{A}$

4. $(\mathrm{A} + \mathrm{B}) + \mathrm{C} = \mathrm{A} + (\mathrm{B} + \mathrm{C})$

5. $\mathrm{A} = \mathrm{B} \cup \mathrm{A} > \mathrm{B} \cup \mathrm{A} < \mathrm{B}$

6. $\mathrm{A} - \varepsilon\, \mathrm{A} + \mathrm{G}$

7. $\mathrm{G} \frown \overline{\mathrm{X}\,\varepsilon}\, (\mathrm{X} < \mathrm{A}) - = \Lambda$

8_1. $\mathrm{A} < \mathrm{B} \,.\, \supset : m \,\varepsilon\, \mathrm{N} \,.\, m\mathrm{A} \overline{\overline{>}} \mathrm{B} \,.\, - =_m \Lambda$

8_2. $n \,\varepsilon\, \mathrm{N} \,.\, \supset .\, \frac{1}{n} \mathrm{A} \,\varepsilon\, \mathrm{G}$

8. $\mathrm{H} \,\varepsilon\, \mathrm{KG} \,.\, \mathrm{H} - = \Lambda : \mathrm{A} \,\varepsilon\, \mathrm{G} \,.\, \mathrm{H} \frown (\mathrm{A} + \mathrm{G}) = \Lambda : \supset .\therefore \mathrm{l'H} \,\varepsilon\, \mathrm{G}$

P.

1. $\mathrm{Pp}1 \,.\, \mathrm{Pp}3 \,.\, \supset .\, \mathrm{Pp}1_1$ (§2P1)

2. $\mathrm{Pp}2 \,.\, \mathrm{Pp}3 \,.\, \supset .\, \mathrm{Pp}2_1$ (§2P2)

3. $\mathrm{Pp}1_1 \,.\, \mathrm{Pp}3 \,.\, \supset .\, \mathrm{Pp}1$ (§2P1′)

4. $\mathrm{Pp}2_1 \,.\, \mathrm{Pp}3 \,.\, \supset .\, \mathrm{Pp}2$ (§2P2′)

5. $\mathrm{Pp}1 \circ \mathrm{Pp}1_1 \,.\, \supset .\, - \mathrm{Pp}3$

6. $\mathrm{Pp}2 \circ \mathrm{Pp}2_1 \,.\, \supset .\, - \mathrm{Pp}3$

5′. $\mathrm{Pp}3 \,.\, \supset .\, (- \mathrm{Pp}1 \,.\, - \mathrm{Pp}1_1) \cup (\mathrm{Pp}1 \,.\, \mathrm{Pp}1_1)$

6′. $\mathrm{Pp}3 \,.\, \supset .\, (- \mathrm{Pp}2 \,.\, - \mathrm{Pp}2_1) \cup (\mathrm{Pp}2 \,.\, \mathrm{Pp}2_1)$

7. $\mathrm{Pp}1 \,.\, \mathrm{Pp}1_1 \,.\, \mathrm{Pp}2 \,.\, \mathrm{Pp}2_1 \,.\, - \mathrm{Pp}3 \,.\, - = \Lambda$

8. $\mathrm{Pp}1 \,.\, \mathrm{Pp}1_1 \,.\, \mathrm{Pp}4 \,.\, \mathrm{Pp}8_2 \,.\, \supset .\, \mathrm{Pp}7$ (§6P2)

9. $\mathrm{Pp}7 \,.\, - \mathrm{Pp}8_2 \,.\, - = \Lambda$

10. $\mathrm{Pp}1 \,.\, \mathrm{Pp}2 \,.\, \mathrm{Pp}3 \,.\, \mathrm{Pp}4 \,.\, \mathrm{Pp}5 \,.\, \mathrm{Pp}6 \,.\, \mathrm{Pp}8 \,.\, \supset .\, \mathrm{Pp}8_1$ (§7P5)

11. $\mathrm{Pp}8_1 \,.\, - \mathrm{Pp}8 \,.\, - = \Lambda$

12. $\mathrm{Pp}1 \,.\, \mathrm{Pp}2 \,.\, \mathrm{Pp}3 \,.\, \mathrm{Pp}4 \,.\, \mathrm{Pp}5 \,.\, \mathrm{Pp}6 \,.\, \mathrm{Pp}7 \,.\, \mathrm{Pp}8 \,.\, \supset .\, \mathrm{Pp}8_2$ (§7P8)

13. $\mathrm{Pp}8_2 \,.\, - \mathrm{Pp}8 \,.\, - = \Lambda$

14. $Pp(1.1_1.2.2_1.3.4.5.6.7.8.8_1.8_2) = \iota Pp1 \cup \iota Pp1_1 \cup \iota Pp2 \cup \iota Pp2_1 \cup \iota Pp3 \cup \iota Pp4 \cup \iota Pp5 \cup \iota Pp6 \cup \iota Pp7 \cup \iota Pp8 \cup \iota Pp8_1 \cup \iota Pp8_2$ (Def.)

15. $f \varepsilon Pp(1.2.3.4.5.6.7.8)/Z_8 . \supset : f1.f2.f3.f4.f5.f6.f7.-f8.-=\Lambda$

15′. $f \varepsilon Pp(1.2_1.3.4.5.6.7.8)/Z_8 . \supset :$ »

15″. $f \varepsilon Pp(1_1.2.3.4.5.6.7.8)/Z_8 . \supset :$ »

15‴. $f \varepsilon Pp(1_1.2_1.3.4.5.6.7.8)/Z_8 . \supset :$ »

16. $f \varepsilon Pp(1.2.3.4.5.6.8_1.8_2)/Z_8 . \supset :$ »

16′. $f \varepsilon Pp(1.2_1.3.4.5.6.8_1.8_2)/Z_8 . \supset :$ »

16″. $f \varepsilon Pp(1_1.2.3.4.5.6.8_1.8_2)/Z_8 . \supset :$ »

16‴. $f \varepsilon Pp(1_1.2_1.3.4.5.6.8_1.8_2)/Z_8 . \supset :$ »

17. $f \varepsilon Pp(1.2.3.4.5.6.7.8_1)/Z_8 . \supset :$ »

17′. $f \varepsilon Pp(1.2_1.3.4.5.6.7.8_1)/Z_8 . \supset :$ »

17″. $f \varepsilon Pp(1_1.2.3.4.5.6.7.8_1)/Z_8 . \supset :$ »

17‴. $f \varepsilon Pp(1_1.2_1.3.4.5.6.7.8_1)/Z_8 . \supset :$ »

18. $f \varepsilon Pp(1.1_1.2.2_1.4.5.6.7.8)/Z_9 . \supset : f1.f2.f3.f4.f5.f6.f7.f8.-f9.-=\Lambda$

19. $f \varepsilon Pp(1.1_1.2.2_1.4.5.6.8_1.8_2)/Z_9 . \supset :$ »

20. $f \varepsilon Pp(1.1_1.2.2_1.4.5.6.7.8_1)/Z_9 . \supset :$ »

§ 2.

$A, B, C, D \varepsilon G . \supset :$

1. $A = B . \supset . C + A = C + B$ [1 .3]

[Hp. $Pp1, 3 : \supset : A+C=B+C . A+C=C+A . B+C=C+B : \supset :$ Ts]

1′. $A = B . \supset . A + C = B + C$ [1_1 .3]

[Hp. $Pp1_1, 3 : \supset : C+A=C+B . C+A=A+C . C+B=B+C : \supset :$ Ts]

2. $C + A = C + B . \supset . A = B$ [2 .3]

[Hp. $Pp3 : \supset : A + C = B + C . Pp2 : \supset :$ Ts]

2′. $A + C = B + C . \supset . A = B$ [2_1 .3]

[Hp. $Pp3 : \supset : C + A = C + B . Pp2_1 : \supset :$ Ts]

3. $A = B . C = D . \supset . A + C = B + D$ [1 . 1_1]

[Hp. $Pp1 . 1_1 : \supset : A + C = B + C . B + C = B + D : \supset :$ Ts]

4. $A = B . A + C = B + D . \supset . C = D$ [1 . 2]

[Hp. $Pp1 : \supset : A+C=B+C .$ Hp $: \supset : B+C=B+D . Pp2_1 : \supset :$ Ts]

5. $C = D . A + C = B + D . \supset . A = B$ [1_1 . 2]

[Hp. $Pp1_1 : \supset : A+C=A+D .$ Hp $: \supset : A+D=B+D . Pp2 : \supset :$ Ts]

6. $n \varepsilon 1 + \mathrm{N} . f \varepsilon \mathrm{G}|\mathrm{Z}_n . \supset . \Sigma_1^n f \varepsilon \mathrm{G}$

[(α) Hp. Pp0 : $\supset$: $2 \varepsilon \overline{n \varepsilon}$ (Ts)

(β) Hp. $m \varepsilon \overline{n \varepsilon}$ (Ts) . $h \varepsilon \mathrm{G}|\mathrm{Z}_{m+1}$. Pp0 : $\supset$: $\Sigma_1^m h + h(m+1) \varepsilon \mathrm{G}$. Pp0 : $\supset$: $\Sigma_1^{m+1} h \varepsilon \mathrm{G}$: $\supset$: $m + 1 \varepsilon \overline{n \varepsilon}$ (Ts)

(α) . (β) . Pi : $\supset$: Ts]

7. $n \varepsilon 1 + \mathrm{N} . f \varepsilon \mathrm{G}|\mathrm{Z}_n . f' \varepsilon \mathrm{G}|\mathrm{Z}_n : s \varepsilon \mathrm{Z}_n . \supset_s . fs = f's : \supset .\therefore \Sigma_1^n f = \Sigma_1^n f'$ [1 . 1₁]

[P3, 6 . Pi : $\supset$: P7]

8. $n \varepsilon 2 + \mathrm{N} . f \varepsilon \mathrm{G}|\mathrm{Z}_n . \supset . \Sigma_1^n f = f1 + \Sigma_2^n f$ [1 . 1₁ . 4]

[(α) Hp. Pp4 : $\supset$: $3 \varepsilon \overline{n \varepsilon}$ (Ts)

(β) Hp. $m \varepsilon \overline{n \varepsilon}$ (Ts) . $h \varepsilon \mathrm{G}|\mathrm{Z}_{n+1}$. Pp1 : $\supset$: $\Sigma_1^{m+1} h = (h1 + \Sigma_2^m h) + h(m+1)$. Pp 4 : $\supset$: $\Sigma_1^{m+1} h = h1 + \left(\Sigma_2^m h + h(m+1)\right)$. Pp1₁ : $\supset$: $\Sigma_1^{m+1} h = h_1 + \Sigma_2^{m+1} h$: $\supset$: $m + 1 \varepsilon \overline{n \varepsilon}$ (Ts).

(α) . (β) . Pi : $\supset$: Ts]

9. $n \varepsilon 2 + \mathrm{N} . f \varepsilon \mathrm{G}|\mathrm{Z}_n . r \varepsilon \mathrm{Z}_{n-1} . \supset . \Sigma_1^n f = \Sigma_1^r f + \Sigma_{r+1}^n f$ [1 . 1₁ . 4]

[(α) Hp. P8 : $\supset$: $1 \varepsilon \overline{r \varepsilon}$ (Ts)

(β) Hp. $r = n - 1$. Def 1 : $\supset$: Ts

(γ) Hp. $r < n - 1$. P8 : $\supset$: $\Sigma_{r+2}^n f = f(r+1) + \Sigma_{r+2}^n f$. Pp 1₁ : $\supset$: $\Sigma_1^r f + \Sigma_{r+1}^n f = \Sigma_1^r f + \left(f(r+1) + \Sigma_{r+2}^n f\right)$. Pp 4 : $\supset$: $\Sigma_1^r f + \Sigma_{r+1}^n f = \left(\Sigma_1^r f + f(r+1)\right) + \Sigma_{r+2}^n f$. Pp 1 : $\supset$: $\Sigma_1^r f + \Sigma_{r+1}^n f = \Sigma_1^{r+1} f + \Sigma_{r+2}^n f$.

(δ) Hp. (γ) : $\supset$: $r < n - 1 . r \varepsilon \overline{r \varepsilon}$ (Ts) . $\supset . r + 1 \varepsilon \overline{r \varepsilon}$ (Ts) .

(α) . (β) . (δ) . Pi : $\supset$: Ts]

10. $n \varepsilon 1 + \mathrm{N} . f \varepsilon \mathrm{G}|\mathrm{Z}_n . r, s \varepsilon \mathrm{Z}_n . \supset . \Sigma_1^n f = \begin{pmatrix} r, s \\ s, r \end{pmatrix} \Sigma_1^n f$ [1 . 3 . 4]

[(α) Hp. Pp3 : $\supset$: $2 \varepsilon \overline{n \varepsilon}$ (Ts).

(β_1) Hp. $m \varepsilon \overline{n \varepsilon}$ (Ts) . $h \varepsilon \mathrm{G}|\mathrm{Z}_{m+1} . r, s \varepsilon \mathrm{Z}_m$. Pp1 : $\supset$: $\Sigma_1^{m+1} h = \begin{pmatrix} r, s \\ s, r \end{pmatrix} \Sigma_1^{m+1} h$

(β_2) $m \varepsilon 1 + \mathrm{N} . h \varepsilon \mathrm{G}|\mathrm{Z}_{m+1}$. P9. Pp3, 1 : $\supset$: $\Sigma_1^{m+1} h = \begin{pmatrix} m, m+1 \\ m+1, m \end{pmatrix} \Sigma_1^{m+1} h$

(β) Hp. $m \varepsilon \overline{n \varepsilon}$ (Ts) . (β_1) . (β_2) : $\supset$: $m + 1 \varepsilon \overline{n \varepsilon}$ (Ts)

(α) . (β) . Pi : $\supset$: Ts]

11. $n \varepsilon 1 + \mathrm{N} . f \varepsilon \mathrm{G}|\mathrm{Z}_n . g \varepsilon (\mathrm{Z}_n|\mathrm{Z}_n)$ sim $. \supset . \Sigma_1^n f = \Sigma_{r=1}^{r=n} f(gr)$ [1 . 3 . 4]

[(α) Hp. Pp3 : $\supset$: $2 \varepsilon \overline{n\,\varepsilon}$ (Ts)

(β_1) Hp. $m \varepsilon \overline{n\,\varepsilon}$ (Ts) . $h \varepsilon G|Z_{m+1}$. $v \varepsilon (Z_{m+1}|Z_{m+1})$ sim . $v(m+1) = m+1$. Pp 1 : $\supset$: $\sum_1^{m+1} h = \sum_{r=1}^{r=m+1} h(vr)$

(β_2) $m \varepsilon 1 + N$. $h \varepsilon G|Z_{m+1}$. $v \varepsilon (Z_{m+1}|Z_{m+1})$ sim . $s \varepsilon Z_{m+1}$. $vs = m+1$. P 10 : $\supset$: $\sum_{r=1}^{r=m+1} h(vr) = \begin{pmatrix} vs, & v(m+1) \\ v(m+1), & vs \end{pmatrix} \sum_{r=1}^{r=m+1} h(vr)$

(β) Hp. $m \varepsilon \overline{n\,\varepsilon}$ (Ts) . (β_1) . (β_2) : $\supset$: $m+1 \varepsilon \overline{n\,\varepsilon}$ (Ts)

(α) . (β) . Pi : $\supset$: Ts]

§ 3.

A, B, C, D ε G . $\supset$:

1. A = B . B > C . $\supset$. A > C

[Hp. Def 1 : $\supset$: B ε C + G . Hp : $\supset$: A ε C + G . Def 1 : $\supset$: Ts]

1'. A = B + C . $\supset$. A > B (Def 1 = P 1')

1''. » . $\supset$. A > C [3]

[Hp. Pp3 : $\supset$: A = C + B . P1' : $\supset$: Ts]

2. A > B . B > C . $\supset$. A > C [1 . 4]

[Hp. Def 1 : $\supset$: A ε B + G . B ε C + G . Pp1 : $\supset$: A ε (C + G) + G . Pp4, 0 : $\supset$: A ε C + G . Def 1 : $\supset$: Ts]

3. A > B . $\supset$. A + C > B + C [1 . 3 . 4]

[Hp. Def 1 : $\supset$: A ε B + G . Pp1 : $\supset$: A + C ε (B + G) + C . Pp1, 3, 4 : $\supset$: A + C ε (B + C) + G . Def 1 : $\supset$: Ts]

4. A > B . $\supset$. C + A > C + B [1_1 . 4]

[Hp. Def 1 : $\supset$: A ε B + G . Pp1_1 : $\supset$: C + A ε C + (B + G) . Pp4 : $\supset$: C + A ε (C + B) + G . Def 1 : $\supset$: Ts]

5. C + A > C + B . $\supset$. A > B [2_1 . 4]

[Hp. : $\supset$: C + A ε (C + B) + G . Pp4 : $\supset$: C + A ε C + (B + G) . Pp2_1 : $\supset$: A ε B + G : $\supset$: Ts]

6. A + C > B + C . $\supset$. A > B [2 . 3 . 4]

[Hp. Pp3 . P1 : $\supset$: C + A > C + B . P5 : $\supset$: Ts]

7. A = B . C > D . $\supset$. A + C > B + D [1 . 1_1 . 4]

[Hp. Pp1 . P4 : $\supset$: A + C = B + C . B + C > B + D . P 1 : $\supset$: Ts]

8. A = B . C > D . $\supset$. C + A > D + B [1 . 3 . 4]

[Hp. Pp1_1 . P3 : $\supset$: C + A = C + B . C + B > D + B . P1 : $\supset$: Ts]

9. $A > B . C > D . \supset . A + C > B + D$ [1 . 3 . 4]
[Hp. $P3, 4 : \supset : A + C > B + C . B + C > B + D . P2 : \supset :$ Ts]

10. $A > B . \supset . A + C > B$ [1 . 4]
[Def 1 $: \supset : A + C > A$. Hp. $P2 : \supset :$ Ts]

11. $A > B + C . \supset . A > B$ [4]
[Hp. $: \supset : A \varepsilon (B + C) + G . Pp0, 4 : \supset : A \varepsilon B + G : \supset :$ Ts]

12. $A > B + C . \supset . A > C$ [3 . 4]
[Hp. $Pp3 . P1 : \supset : A > C + B . P11 : \supset :$ Ts]

13. $A + B = C + D . A > C . \supset . B < D$ [1 . 2 . 3 . 4]
[Hp. $: \supset : A \varepsilon C + G . Pp1 . Hp : \supset : C + D \varepsilon (C + G) + B . Pp4, 2_1$
$: \supset : D \varepsilon G + B . Pp3 .$ Def 1, 2 $: \supset :$ Ts]

21. $A -= B . \supset . A > B \cup A < B$ [5]
[I §2P25 $: \supset :$ P21 $=$ Pp5]

22. $A -> B . \supset . A = B \cup A < B$ [5]

23. $A -< B . \supset . A = B \cup A > B$ [5]

24. $A = B . \supset . A -> B . A -< B$ [6]
[Hp. $(A > B \cup A < B) . P1 : \supset : B > B .$ Def 1 $: \supset : B \varepsilon B + G . Pp6 .$ I
§3P1 $: \supset .\therefore A = B . (A > B \cup A < B) : = \Lambda .$ I §3P8 . §2P7 $.\therefore \supset :$ P24]

25. $A > B . \supset . A -= B . A -< B$ [6]

26. $A < B . \supset . A -= B . A -> B$ [6]

27. $A -= B . = . A > B \cup A < B$ [5 . 6]
[P21, 24 . I §2P1, §1P3 $: \supset :$ P27]

28. $A -> B . = . A = B \cup A < B$ [5 . 6]

29. $A -< B . = . A = B \cup A > B$ [5 . 6]

31. $G -= \Lambda . \supset . \mathrm{num}\, G = \infty$ [6]
[(α) Hp. $. \supset . A \varepsilon G . -=_{\Lambda} \Lambda$
(β) $A \varepsilon G . Pp0 : \supset : A + A \varepsilon G$
(γ) $A \varepsilon G . Pp6 : \supset : A -= A + A$
Hp. $n \varepsilon N . \mathrm{num}\, G \overset{=}{>} n . (\alpha) . (\beta) . (\gamma) : \supset : \mathrm{num}\, G \overset{=}{>} n+1 .$ Pi $: \supset :$ Ts]

32. $G -= \Lambda . \supset . \mathrm{num}\, (A + G) = \infty$ [6]

§ 4.

$A, B, C, D \,\varepsilon\, G . \supset :$

1. $A > B . \supset . A - B \,\varepsilon\, G$ $[2_1]$

$[(\alpha)$ Hp. $\S 3$ Def $1 : \supset : X \,\varepsilon\, G . A = B + X . - =_x \Lambda$

(β) $X, Y \,\varepsilon\, G . A - B = X . A - B = Y .$ Def $1 :\supset: A = B + X . A = B + Y$
$: \supset : B + X = B + Y .$ $\mathrm{Pp}2_1 : \supset : X = Y$

$(\alpha) . (\beta) : \supset .\!\cdot\!. A > B . A - B - \varepsilon\, G : = \Lambda .$ I $\S 3$P8 $: \supset :$ P 1$]$

2. $A - B \,\varepsilon\, G . \supset . A > B$

[Hp. Def $1 : \supset : A = B + (A - B) .$ Hp. $\S 3$ Def $1 : \supset :$ Ts]

2'. $A \geq B . = . A - B \,\varepsilon\, G$ $[2_1]$

3. $A - B - \varepsilon\, G . \supset . A \overset{=}{<} B$ $[2_1 . 5]$

[P1 . I $\S 2$P4 $: \supset : A - B - \varepsilon\, G . \supset . A -> B .$ $\S 3$P22 $: \supset :$ P 3]

4. $A \overset{=}{<} B . \supset . A - B - \varepsilon\, G$ $[5 . 6]$

[P2 . $\S 3$P28 . I $\S 2$P4 $: \supset :$ P4]

5. $A > B . \supset . A = B + (A - B)$ $[2_1]$

[Hp. P1 $: \supset : A - B \,\varepsilon\, G .$ Def $1 : \supset :$ Ts]

6. $A > B . \supset . A = A - B + B$ $[2 . 3]$

6'. $A > B . \supset . B = A - (A - B)$ $[2 . 3]$

[Hp. P5 . Pp3 $: \supset : A = (A - B) + B .$ Def $1 : \supset :$ Ts]

7. $A = A + B - B$ $[3]$

[Pp3 $: \supset : B + A = A + B .$ Def $1 : \supset : A = (A + B) - B .$ Def $3 : \supset :$ Ts]

8. $A = B . A > C . \supset . A - C = B - C$ $[2_1]$

[Hp. $\S 3$P1 $: \supset : B > C .$ Hp. P5 $: \supset : A = C + (A - C) . B = C + (B - C) .$
Hp. $: \supset : C + (A - C) = C + (B - C) .$ $\mathrm{Pp}2_1 : \supset :$ Ts]

9. $A = B . C > A . \supset . C - A = C - B$ $[1 . 2_1]$

[Hp. $\S 3$P1 $: \supset : C > B .$ Hp. P5 $: \supset : C = A + (C - A) . C = B + (C - B)$
$: \supset : A + (C - A) = B + (C - B) .$ Hp. $\S 2$P4 $: \supset :$ Ts]

10. $A = B . C = D . A > C . \supset . A - C = B - D$ $[1 . 2_1]$

[Hp. $\S 3$P1 $: \supset : B > C . B > D .$ P8, 9 $: \supset : A - C = B - C . B - C =$
$B - D : \supset :$ Ts]

11. $A > C . B > C . A - C = B - C . \supset . A = B$ $[1_1 . 2_1]$

[Hp. P1 . $\mathrm{Pp}1_1 : \supset : C + (A - C) = C + (B - C) .$ P5 $: \supset :$ Ts]

12. $C > A . C > B . C - A = C - B . \supset . A = B$ $[2 . 2_1]$

[Hp. P5 $: \supset : C = A + (C - A) . C = B + (C - B) : \supset : A + (C - A)$
$= B + (C - B) .$ Hp. P1 . Pp2 $: \supset :$ Ts]

13. $A > B > C . \supset . A - C > B - C$ [$1 . 2_1 . 4$]

[Hp. §3P2 $: \supset : A > C$. Hp. P5 $: \supset : A = C + (A - C) . B = C + (B - C)$.
Hp. §3P1 $: \supset : C + (A - C) > C + (B - C)$. P1 . §3P5 $: \supset :$ Ts]

14. $A > C . B > C . A - C > B - C . \supset . A > B$ [$1_1 . 2_1 . 4$]

[Hp. P1 . §3P4 $: \supset : C + (A - C) > C + (B - C)$. Hp. P5 $: \supset :$ Ts]

15. $A > B . C > A . \supset . C - A < C - B$ [$1 . 2 . 3 . 4$]

[Hp. §3P2 $: \supset : C > B$. Hp. P5 $: \supset : A + (C - A) = B + (C - B)$.
Hp. §3P13 $: \supset :$ Ts]

16. $C > A . C > B . C - A > C - B . \supset . A < B$ [$1 . 2 . 3 . 4$]

[Hp. P6 $: \supset : (C - A) + A = (C - B) + B$. Hp. §3P13 $: \supset :$ Ts]

17. $A - B = C . \supset . A = C + B$ [$1 . 2 . 3$]

[(α) Hp. P2 $: \supset : A > B$
Hp. Pp1 $: \supset : (A - B) + B = C + B$. Pp3 $: \supset : B + (A - B) = C + B$. (α) . P5 $: \supset :$ Ts]

18. $A - B = C . \supset . A = B + C$ [$1_1 . 2_1$]

[(α) Hp. P2 $: \supset : A > B$
Hp. Pp1_1 $: \supset : B + (A - B) = B + C$. (α) . P5 $: \supset :$ Ts]

19. $A - B > C . \supset . A > C + B$ [$1 . 2 . 3 . 4$]

20. » $. \supset . A > B + C$ [$1_1 . 2_1 . 4$]

21. $A > B + C . \supset . A - B > C$ [$1 . 2_1 . 4$]

[Hp. §3 Def 1 $: \supset : B + C > B$. Hp. P13 $: \supset : A - B > (B + C) - B$ Def 1.
§3P1 $: \supset :$ Ts]

22. $A > B + C . \supset . A - C > B$ [$1 . 2 . 3 . 4$]

23. $A < B + C . A > B . \supset . A - B < C$ [$1 . 2_1 . 4$]

23'. $A < B + C . A > C . \supset . A - C < B$ [$1 . 2 . 3 . 4$]

24. $B > C . \supset . A + (B - C) = A + B - C$ [$1 . 2 . 3 . 4$]

[Hp. P6 $: \supset : (B - C) + C = B$. Pp1_1 $: \supset : A + \big((B - C) + C\big) = A + B$.
Pp4 $: \supset : \big(A + (B - C)\big) + C = A + B$. Pp3 . Def 1 $: \supset :$ Ts]

25. $B > C . A > B - C . \supset . A - (B - C) = A + C - B$ [$1 . 2 . 3 . 4$]

26. $A > B . B > C . \supset . A - (B - C) = A - B + C$ [$1 . 2 . 3 . 4$]

27. $A > B + C . \supset . A - (B + C) = A - B - C$ [$1 . 2 . 3 . 4$]

28. $A > B . \supset . A - B < A$ [$2 . 3$]

[Hp. P6 $: \supset : (A - B) + B = A$. Hp. P1 . §3 Def 1 $: \supset :$ Ts]

29. $A > B . B > D . D > C . \supset . A - C > B - D$ [$1 . 2 . 3 . 4$]

[Hp. §3P2 . §4P13 $: \supset : A - C > B - C$. Hp. §3P9 $: \supset : (A - C) +$
$D > (B - C) + D$. P6.§3P1 $: \supset : (A - C) + D > B$. Hp. P23 $: \supset :$ Ts]

31. $A \varepsilon G . \supset . G \frown \overline{X \varepsilon} (X < A) = G \frown (A - G)$ [2 . 3]

[(α) Hp. $B \varepsilon G \frown (A - G)$. P2, 28 : $\supset$: $B < A$

(β_1) » $B \varepsilon G \frown \overline{X \varepsilon} (X < A)$. P6' :$\supset$: $B = A - (A - B)$.P1 :$\supset$: $B \varepsilon A - G$

» (α) . (β) . I §4P2 : $\supset$: Ts]

§ 5.

A, B, C ε G . m, $n \varepsilon$ N . $\supset$:

1. $nA \varepsilon G$

[(α) Hp. Def 1 : $\supset$: $1 \varepsilon \overline{n \varepsilon}$ (Ts)

(β) » $n \varepsilon \overline{n \varepsilon}$ (Ts) . P0 : $\supset$: $nA + A \varepsilon G$. Def 2 : $\supset$: $(n+1)A \varepsilon G$:

$\supset$: $n + 1 \varepsilon \overline{n \varepsilon}$ (Ts)

Hp. (α) . (β) . Pi : $\supset$: Ts]

1'. NG = G

[Def 1 . P1 : $\supset$: G $\supset$ NG . NG $\supset$ G . I §4P2 : $\supset$: P1']

2. $f \varepsilon G|Z_n : s \varepsilon Z_n . \supset_s . fs = A : \supset .\!\cdot\!. \Sigma_1{}^n f = nA$ [1 . 1_1]

[(α) Hp. Def 1 : $\supset$: $1 \varepsilon \overline{n \varepsilon}$ (Ts)

(β) » $m \varepsilon \overline{n \varepsilon}$ (Ts) . $h \varepsilon G|Z_{m+1} : s \varepsilon Z_{m+1} . \supset_s . hs = A$: P1 . §2P3

$.\!\cdot\!. \supset .\!\cdot\!. \Sigma_1^{m+1} h = mA + A$. Def 2 $.\!\cdot\!. \supset .\!\cdot\!. \Sigma_1^{m+1} h = (m+1) A$

$.\!\cdot\!. \supset .\!\cdot\!. m + 1 \varepsilon \overline{n \varepsilon}$ (Ts)

Hp. (α) . (β) . Pi : $\supset$: Ts]

3. $A = B . \supset . nA = nB$ [1 . 1_1]

[(α) Hp. Def 1 : $\supset$: $1 \varepsilon \overline{n \varepsilon}$ (Ts)

(β) » $m \varepsilon \overline{n \varepsilon}$ (Ts) . §2P3 . Def 2 : $\supset$: $(m+1)A = (m+1)B$. Tn

: $\supset$: $m + 1 \varepsilon \overline{n \varepsilon}$ (Ts)

Hp. (α) . (β) . Pi : $\supset$: Ts]

4. $m = n . \supset . mA = nA$ [1]

[(α) Hp. Tn . Def 1 : $\supset$: $(1, 1) \varepsilon \overline{(m, n) \varepsilon}$ (Ts)

(β) » $(m', n') \varepsilon \overline{(m, n) \varepsilon}$ (Ts) . Pp1 . Def 2 :$\supset$:$(m'+1)A=(n'+1)A$.

Tn : $\supset$: $(m'+1, n'+1) \varepsilon \overline{(m, n) \varepsilon}$ (Ts)

Hp. (α) . (β) . Pi : $\supset$: Ts]

5. $A = B . m = n . \supset . mA = nB$ [1 . 1_1]

6. $m(A + B) = mA + mB$ [1 . 3 . 4]

[(α) Hp. §2P3 . Def 1 : $\supset$: $1 \,\varepsilon\, \overline{m\,\varepsilon}$ (Ts)

(β) » $n \,\varepsilon\, \overline{m\,\varepsilon}$ (Ts) . Pp1 : $\supset$: $n(\mathrm{A}+\mathrm{B}) + (\mathrm{A}+\mathrm{B}) = n\mathrm{A} + n\mathrm{B} + (\mathrm{A}+\mathrm{B})$. §2P10, 11 : $\supset$: $n(\mathrm{A}+\mathrm{B}) + (\mathrm{A}+\mathrm{B}) = (n\mathrm{A}+\mathrm{A}) + (n\mathrm{B}+\mathrm{B})$. Def 2 . §2P3 : $\supset$: $(n+1)(\mathrm{A}+\mathrm{B}) = (n+1)\mathrm{A} + (n+1)\mathrm{B}$: $\supset$: $n+1 \,\varepsilon\, \overline{m\,\varepsilon}$ (Ts)

Hp. (α) . (β) . Pi : $\supset$: Ts]

6'. $f \,\varepsilon\, \mathrm{G}|\mathrm{Z}_n \,.\supset.\, m(\Sigma_1^n f) = \Sigma_1^n(mf)$ [1 . 3 . 4]

7. $(m+n)\mathrm{A} = m\mathrm{A} + n\mathrm{A}$ [1 . 1_1 . 4]

[(α) Hp. Def 1, 2 . $\mathrm{Pp}1_1$: $\supset$: $(m+1)\mathrm{A} = m\mathrm{A} + 1\mathrm{A}$: $\supset$: $1 \,\varepsilon\, \overline{n\,\varepsilon}$ (Ts)

(β) » $n' \varepsilon\, \overline{n\,\varepsilon}$ (Ts) . Pp1 : $\supset$: $(m+n')\mathrm{A} + \mathrm{A} = (m\mathrm{A}+n'\mathrm{A}) + \mathrm{A}$. $\mathrm{Pp}1_1$, 4 . Def 2 . Tn . P4 : $\supset$: $\big(m+(n'+1)\big)\mathrm{A} = m\mathrm{A} + (n'+1)\mathrm{A}$: $\supset$: $n'+1 \,\varepsilon\, \overline{n\,\varepsilon}$ (Ts)

Hp. (α) . (β) . Pi : $\supset$: Ts]

7'. $f \,\varepsilon\, \mathrm{N}|\mathrm{Z}_n \,.\supset.\, (\Sigma_1^n f)\mathrm{A} = \Sigma_{r=1}^{r=n}(fr)\mathrm{A}$ [1 . 1_1 . 4]

8. $m(n\mathrm{A}) = (mn)\mathrm{A}$ [1 . 3 . 4]

[(α) Hp. Tn . Def 1 : $\supset$: $1\mathrm{A} = \mathrm{A} \,.\, m = m \times 1$. P5 : $\supset$: $m(1\mathrm{A}) = (m \times 1)\mathrm{A}$: $\supset$: $1 \,\varepsilon\, \overline{n\,\varepsilon}$ (Ts)

(β) Hp. $n' \varepsilon\, \overline{n\,\varepsilon}$ (Ts) . Pp1 . P1 : $\supset$: $m(n'\mathrm{A}) + m\mathrm{A} = (mn')\mathrm{A} + m\mathrm{A}$. P6, 7 : $\supset$: $m(n'\mathrm{A}+\mathrm{A}) = (mn'+m)\mathrm{A}$. P1, 3, 4 . Def 2 . Tn : $\supset$: $m\big((n'+1)\mathrm{A}\big) = \big(m(n'+1)\big)\mathrm{A}$: $\supset$: $n' \varepsilon\, \overline{n\,\varepsilon}$ (Ts)

Hp. (α) . (β) . Pi : $\supset$: Ts]

9. $\mathrm{A} > \mathrm{B} \,.\supset.\, m\mathrm{A} > m\mathrm{B}$ [1 . 3 . 4]

[Hp. : $\supset$: $\mathrm{A} \,\varepsilon\, \mathrm{B} + \mathrm{G}$. Hp. P1, 3, 6 : $\supset$: $m\mathrm{A} \,\varepsilon\, m\mathrm{B} + \mathrm{G}$: $\supset$: Ts]

10. $m > n \,.\supset.\, m\mathrm{A} > n\mathrm{A}$ [1 . 1_1 . 4]

[Hp. Tn . $\supset$. $m \,\varepsilon\, n + \mathrm{N}$. Hp. P1, 4, 7 : $\supset$: $m\mathrm{A} \,\varepsilon\, n\mathrm{A} + \mathrm{G}$: $\supset$: Ts]

11. $\mathrm{A} > \mathrm{B} \,.\, m = n \,.\supset.\, m\mathrm{A} > n\mathrm{B}$ [1 . 3 . 4]

[Hp. P9, 4 : $\supset$: $m\mathrm{A} > m\mathrm{B} \,.\, m\mathrm{B} = n\mathrm{B}$. P1 . §3 P1 : $\supset$: Ts]

12. $\mathrm{A} = \mathrm{B} \,.\, m > n \,.\supset.\, m\mathrm{A} > n\mathrm{B}$ [1 . 1_1 . 4]

[Hp. P3, 10 : $\supset$: $m\mathrm{A} = m\mathrm{B} \,.\, m\mathrm{B} > n\mathrm{B}$. §3P1 : $\supset$: Ts]

13. $\mathrm{A} > \mathrm{B} \,.\, m > n \,.\supset.\, m\mathrm{A} > n\mathrm{B}$ [1 . 3 . 4]

14. $m\mathrm{A} = m\mathrm{B} \,.\supset.\, \mathrm{A} = \mathrm{B}$ [1 . 3 . 4 . 5 . 6]

[A, B ε G . $m \,\varepsilon\,$ N . A $-\!=$ B . §3P27 . P9, 1 : $\supset$: $m\mathrm{A} -\!= m\mathrm{B}$. I §2P1 .:. $\supset$.:. P14]

15. $m\mathrm{A} > m\mathrm{B} \,.\supset.\, \mathrm{A} > \mathrm{B}$ [1 . 3 . 4 . 5 . 6]

16. $m\mathrm{A} = n\mathrm{A} \,.\supset.\, m = n$ [1 . 1_1 . 4 . 5 . 6]

[$A \varepsilon G . m, n \varepsilon N . m -= n . Tn . §3P27 . P10 : \supset : mA -= nA . I §2$ $P1 .\therefore \supset .\therefore P16$]

17. $mA > nA . \supset . m > n$ **[1 . 1_1 . 4 . 5 . 6]**

18. $A > B . \supset . m(A - B) = mA - mB$ **[1 . 2 . 3 . 4]**

[$Hp.\ §4P5 . P3, 6 : \supset : mA = mB + m(A - B) . §4 Def 1 : \supset :\ Ts$]

19. $m > n . \supset . (m - n)A = mA - nA$ **[1 . 1_1 . 4]**

[$Hp.\ Tn : \supset : m = n + (m - n) . P4, 7 : \supset : mA = nA + (m - n)A .$ $§4 Def 1 : \supset :\ Ts$]

20. $p \varepsilon N . (mp)A = (np)B . \supset . mA = nB$ **[1 . 3 . 4 . 5 . 6]**

21. » $. (mp)A > (np)B . \supset . mA > nB$ **[1 . 3 . 4 . 5 . 6]**

22. $n \varepsilon 1 + N . \supset . nA > A$ **[1 . 1_1 . 4]**

[$Hp.\ Tn . \supset . n > 1 . P12 . Def 1 . §3P1 : \supset :\ Ts$]

23. $A = B . \supset . m \varepsilon N . mA \stackrel{=}{>} B . -=_m \Lambda$ **[1 . 1_1 . 4]**

[$Hp.\ Def 1 . P22 . §3P1 . Tn : \supset :\ Ts$]

24. $A > B . \supset . m \varepsilon N . mA > B . -=_m \Lambda$ **[1 . 1_1 . 4]**

[$Hp.\ P22 . §3P2 . Tn : \supset :\ Ts$]

25. $x \varepsilon R . n, nx \varepsilon N . nA = (nx)B . \supset : m, mx \varepsilon N . \supset_m . mA = (mx)B$ **[1.3.4.5.6]**

[$Hp.\ m, mx \varepsilon N . P3, 4, 8 . Tr : \supset : n(mA) = n\big((mx)B\big) . P20 : \supset :\ Ts$]

31. $mA \stackrel{=}{>} B . -=_m \Lambda$ **[1 . 1_1 . 4 . 5 . 8_1]**

[(α) $Hp.\ A \stackrel{=}{>} B . P23, 24 : \supset :\ Ts$

(β) » $A < B . Pp8_1 : \supset :\ Ts$

» $(\alpha) . (\beta) . Pp5 : \supset :\ Ts$]

32. $A \stackrel{=}{>} B . \supset : m \varepsilon N . (m+1)B > A \stackrel{=}{>} mB . -=_m \Lambda$ **[1 . 1_1 . 4 . 5 . 6 . 8_1]**

[(α) $Hp.\ P23, 24 : \supset : N \frown \overline{x \varepsilon}\, (xB \stackrel{=}{<} A) -= \Lambda . P17, 31 . §3P1, 2 . Tn$ $: \supset : \max\big(N \frown \overline{x \varepsilon}\, (xB \stackrel{=}{<} A)\big) \varepsilon N$

(β) $Hp.\ m = \max\big(N \frown \overline{x \varepsilon}\, (xB \stackrel{=}{<} A)\big) . §3P28 : \supset : (m + 1)B > A .$

» $(\alpha) . (\beta) : \supset :\ Ts$]

33. $U \varepsilon G . A < B . \supset : x, y \varepsilon N . xA < yU < xB . -=_{x, y} \Lambda$ **[1.3.4.5.6.8_1]**

[(α) $Hp.\ §4P1 . P31 : \supset : m \varepsilon N . m(B - A) > U . -=_m \Lambda$

(β) » $m \varepsilon N . m(B - A) > U . mA < U . P18 . §4P19 . §3P12 : \supset :$ $U < mB : \supset : (m, 1) \varepsilon \overline{(x, y) \varepsilon}\ (Ts)$

(γ) $Hp.\ m \varepsilon N . mA \stackrel{=}{>} U . P32 : \supset : n \varepsilon N . (n + 1)U \stackrel{=}{>} mA > nU .$ $-=_n \Lambda$

(δ) Hp. $m, n \,\varepsilon\, \mathrm{N} \,.\, m(\mathrm{B}-\mathrm{A}) > \mathrm{U} \,.\, m\mathrm{A} \overset{=}{>} \mathrm{U} \,.\, (n+1)\mathrm{U} > m\mathrm{A} \overset{=}{>} n\mathrm{U}$.
§3P9 . §4P19, 6 : $\supset$: $m\mathrm{A} < (n+1)\,\mathrm{U} < m\mathrm{B}$: $\supset$: $(m, n+1)$
$\varepsilon\, \overline{(x, y)\,\varepsilon}$ (Ts)
Hp. (α) . (β) . (γ) . (δ) . Pp5 : $\supset$: Ts]

34. $\mathrm{G} \frown \overline{\mathrm{X}\,\varepsilon}\, (\mathrm{X} < \mathrm{A}) = \Lambda \,.\, \supset .\, \mathrm{G} = \mathrm{NA}$ **[1 . 3 . 4 . 5 . 6 . 8_1]**
[(α) Hp. $\mathrm{B} \,\varepsilon\, \mathrm{G}$. §3P23 : $\supset$: $\mathrm{B} \overset{=}{>} \mathrm{A}$. P32 : $\supset$: $m \,\varepsilon\, \mathrm{N} \,.\, (m+1)\mathrm{A} > \mathrm{B}$
$\overset{=}{>} m\mathrm{A} \,.\, -=_m \Lambda$
(β) Hp. $\mathrm{B} \,\varepsilon\, \mathrm{G} \,.\, m \,\varepsilon\, \mathrm{N} \,.\, \mathrm{B} > m\mathrm{A}$. §3P23 : $\supset$: $\mathrm{B} - m\mathrm{A} \overset{=}{>} \mathrm{A}$. §4P18,
19 : $\supset$: $\mathrm{B} \overset{=}{>} (m+1)\mathrm{A}$
Hp. (α) . (β) . §3P29 : $\supset$: $\mathrm{B} \,\varepsilon\, \mathrm{G} \,.\, \mathrm{B} -\varepsilon\, \mathrm{NA} \,.= \Lambda$. I §3P8 : $\supset$:
$\mathrm{B} \,\varepsilon\, \mathrm{G} \,.\supset.\, \mathrm{B} \,\varepsilon\, \mathrm{NA}$. P1′. I §4P2 : $\supset$: Ts]

35. $\mathrm{G} \frown (-\mathrm{NA}) -= \Lambda \,.\supset.\, \mathrm{G} \frown \overline{\mathrm{X}\,\varepsilon}\, (\mathrm{X} < \mathrm{A}) -= \Lambda$ **[1 . 3 . 4 . 5 . 6 . 8_1]**
[(α) P34 . §4P31 . I §2P1 : $\supset$: $\mathrm{G} -= \mathrm{NA} \,.\supset.\, \mathrm{G} \frown (\mathrm{A} - \mathrm{G}) -= \Lambda$
(β) P1′. I §4P2 . §2P2 : $\supset$: $\mathrm{G} -= \mathrm{NA} \,.=.\, \mathrm{G} \frown (-\mathrm{NA}) -= \Lambda$
(α) . (β) . I §1P21 : $\supset$: P35]

36. $\mathrm{G} \frown \overline{\mathrm{X}\,\varepsilon}\, (\mathrm{X} < \mathrm{A}) -= \Lambda \,.\supset.\, \mathrm{G} \frown (-\mathrm{NA}) -= \Lambda$ **[1 . 2 . 3 . 4 . 6]**
[(α) Hp. §4P31 : $\supset$: $\mathrm{B} \,\varepsilon\, \mathrm{G} \frown (\mathrm{A} - \mathrm{G}) \,.\, -=_{\mathrm{B}} \Lambda$
(β) » $\mathrm{B} \,\varepsilon\, \mathrm{G} \frown (\mathrm{A} - \mathrm{G}) \,.\, \mathrm{B} \,\varepsilon\, \mathrm{NA}$. Def 1 . P22 : $\supset$. $\mathrm{B} \overset{=}{>} \mathrm{A}$
(γ) » §4P28 . §3P26 : $\supset$: $\mathrm{B} \,\varepsilon\, \mathrm{G} \frown (\mathrm{A} - \mathrm{G}) \,.\, \mathrm{B} \,\varepsilon\, \mathrm{NA} \,.= \Lambda$
(α) . (γ) . I §3P8 : $\supset$: P36]

41. $\mathrm{A} < \mathrm{B} + \mathrm{C} \,.\supset:\, \mathrm{X}, \mathrm{Y} \,\varepsilon\, \mathrm{G} \,.\, \mathrm{X} + \mathrm{Y} = \mathrm{A} . \mathrm{X} < \mathrm{B} . \mathrm{Y} < \mathrm{C} \,.\, -=_{\mathrm{X}, \mathrm{Y}} \Lambda$ **[1.2.3.4.5.7]**
[(α) Hp. $\mathrm{A} \overset{=}{<} \mathrm{B} \,.\, \mathrm{D} \,\varepsilon\, \mathrm{G} \frown (\mathrm{A} - \mathrm{G}) \frown (\mathrm{C} - \mathrm{G})$. §4P6, 28 . §3P1, 2 : $\supset$:
$(\mathrm{A} - \mathrm{D}, \mathrm{D}) \,\varepsilon\, \overline{(\mathrm{X}, \mathrm{Y})\,\varepsilon}$ (Ts)
(β) Hp. $\mathrm{A} > \mathrm{B}$. §4P23, 1 : $\supset$: $\mathrm{C} - (\mathrm{A} - \mathrm{B}) \,\varepsilon\, \mathrm{G}$
(γ) » $\mathrm{A} > \mathrm{B} \,.\, \mathrm{E} \,\varepsilon\, \mathrm{G} \frown \big(\mathrm{C} - (\mathrm{A} - \mathrm{B}) - \mathrm{G}\big) \frown (\mathrm{B} - \mathrm{G})$. §4P19, 28, 6, 7
: $\supset$: $\big(\mathrm{B} - \mathrm{E}, \mathrm{E} + (\mathrm{A} - \mathrm{B})\big) \,\varepsilon\, \overline{(\mathrm{X}, \mathrm{Y})\,\varepsilon}$ (Ts)
(α_1) Hp. Pp7, 5 . §3P1, 2 : $\supset$: $\mathrm{D} \,\varepsilon\, \mathrm{G} \frown (\mathrm{A} - \mathrm{G}) \frown (\mathrm{C} - \mathrm{G}) \,.\, -=_{\mathrm{D}} \Lambda$
(γ_1) » $\mathrm{A} > \mathrm{B}$. (β) . Pp7, 5 . §3P1, 2 : $\supset$: $\mathrm{E} \,\varepsilon\, \mathrm{G} \frown \big(\mathrm{C} - (\mathrm{A} - \mathrm{B}) - \mathrm{G}\big)$
$\frown (\mathrm{B} - \mathrm{G}) \,.\, -=_{\mathrm{E}} \Lambda$
Hp. (α) . (α_1) . (γ) . (γ_1) . Pp5 : $\supset$: Ts]

42. $\mathrm{A} < \mathrm{B} \,.\supset:\, \mathrm{X} \,\varepsilon\, \mathrm{G} \,.\, \mathrm{A} < \mathrm{X} < \mathrm{B} \,.\, -=_{\mathrm{X}} \Lambda$ **[1_1 . 2_1 . 4 . 7]**
[(α) Hp. $\mathrm{C} \,\varepsilon\, \mathrm{G} \,.\, \mathrm{C} < \mathrm{B} - \mathrm{A}$. §4P20 : $\supset$: $\mathrm{A} + \mathrm{C} \,\varepsilon\, \overline{\mathrm{X}\,\varepsilon}$ (Ts)
(β) » §4P1 : $\supset$: $\mathrm{B} - \mathrm{A} \,\varepsilon\, \mathrm{G}$. Pp7 : $\supset$: $\mathrm{C} \,\varepsilon\, \mathrm{G} \frown (\mathrm{B} - \mathrm{A} - \mathrm{G}) \,.\, -=_{\mathrm{C}} \Lambda$
» (α) . (β) : $\supset$: Ts]

43. $X, A - X \,\varepsilon\, G \,.\, A - X < B \,.\, -=_{x} \Lambda$ **[1 . 2 . 3 . 4 . 5 . 7]**

[(α) Hp. $A \leqq B \,.\, C \,\varepsilon\, G \frown (A - G)$. §4P28 . §3P1, 2 : ɔ : $C \,\varepsilon\, \overline{X\,\varepsilon}$ (Ts)

(β) » $A > B \,.\, D \,\varepsilon\, G \,.\, A - B < D < A$. §4P20, 23′ : ɔ : $D \,\varepsilon\, \overline{X\,\varepsilon}$ (Ts)

(α_1) » Pp7 : ɔ : $C \,\varepsilon\, G \frown (A - G) \,.\, -=_{c} \Lambda$

(β_1) » $A > B$. §4P1, 28 . P42 : ɔ : $D \,\varepsilon\, G \,.\, A - B < D < A \,.\, -=_{D} \Lambda$

» (α) . (α_1) . (β) . (β_1) . Pp5 : ɔ : Ts]

43′. $A \geqq B$. ɔ : $X, A - X \,\varepsilon\, G \,.\, A - X < B \,.\, -=_{x} \Lambda$

43″. $A \leqq B$. ɔ : » **[1 . 2 . 3 . 4 . 7]**

43‴. $A \gtrless B$. ɔ : »

44. $f \,\varepsilon\, G|Z_n \,.\, \Sigma_1^n f \leqq A \,.\, -=_{f} \Lambda$ **[1_1 . 2_1 . 4 . 7]**

[(α) Hp. Pp7 . I §4P7 : ɔ : $1 \,\varepsilon\, \overline{n\,\varepsilon}$ (Ts)

(β_1) » $B \,\varepsilon\, G$. Pp7 . §4P5 : ɔ : $U, V \,\varepsilon\, G \,.\, U + V = B \,.\, -=_{U, V} \Lambda$

(β_2) » $m \,\varepsilon\, \overline{n\,\varepsilon}$ (Ts) . (β_1) . Pp 1_1 . Pp 4 . § 3 P 1 : ɔ : $h \,\varepsilon\, G|Z_{m+1}$.
$\Sigma_1^{m+1} h \leqq A \,.\, -=_{h} \Lambda$

(β) Hp. (β_2) : ɔ : $m \,\varepsilon\, \overline{n\,\varepsilon}$ (Ts) . ɔ . $(m+1) \,\varepsilon\, \overline{n\,\varepsilon}$ (Ts)

» (α) . (β) . Pi : ɔ : Ts]

45. $X \,\varepsilon\, G \,.\, nX \leqq A \,.\, -=_{x} \Lambda$ **[1 . 2 . 3 . 4 . 5 . 7]**

[(α) Hp. Pp5 . §3P1, 2 . P44 : ɔ :: $f \,\varepsilon\, G|Z_n \,.\, \Sigma_1^n f \leqq A : r \,\varepsilon\, Z_n$. ɔ$_r$.
$f1 < fr : -=_{f} \Lambda$

(β) Hp. $f \,\varepsilon\, G|Z_n : r \,\varepsilon\, Z_n$. ɔ$_r$. $f1 \leqq fr$: §3P9 . §5P2.·.ɔ:$n(f1) \leqq \Sigma_1^n f$

» (α) . (β) . §3P2 : ɔ : Ts]

47. $A < B + C$. ɔ : $m, n \,\varepsilon\, N \,.\, m > n \,.\, nA < mB \,.\, (m - n)A < mC \,.\, -=_{m,n} \Lambda$
[1 . 3 . 4 . 5 . 6 . 7 . 8_1]

[(α) Hp. $A \leqq B \,.\, A \leqq C$. P13 . Tn : ɔ : Ts

(β_1) » §4P1 . P7 : ɔ : $D \,\varepsilon\, G \,.\, D < B + C - A \,.\, D < B \,.\, -=_{D} \Lambda$

(β_2) » $D \,\varepsilon\, G \frown (B - G)$. §4P28 . P33 : ɔ : $m, n \,\varepsilon\, N \,.\, m(B - D) <$
$nA < mB -=_{m,n} \Lambda$

(β_3) Hp. $m, n \,\varepsilon\, N \,.\, nA < mB \,.\, A > B$. P13, 17 : ɔ : $n < m$

(β_4) » $D \,\varepsilon\, G \,.\, D < B + C - A \,.\, D < B \,.\, m, n \,\varepsilon\, N \,.\, m > n . m(B - D)$
$< nA$: ɔ : $(m - n)A < mC$

(β) Hp. $A > B$. (β_1) . (β_2) . (β_3) . (β_4) : ɔ : Ts

» (α) . (β) . Pp5 : ɔ : Ts]

48. $x \,\varepsilon\, \mathrm{R} \,.\, n, nx \,\varepsilon\, \mathrm{N} \,.\, (nx)\mathrm{A} < n\mathrm{B} + n\mathrm{C} \,.\, \supset : y, z \,\varepsilon\, \mathrm{R} \,.\, m, my, mz \,\varepsilon\, \mathrm{N} \,.\, x = y + z \,.\, (my)\mathrm{A} < m\mathrm{B} \,.\, (mz)\mathrm{A} < m\mathrm{C} \,.\, -=_{m,y,z} \Lambda$

[$1 . 3 . 4 . 5 . 6 . 7 . 8_1$]

49. $\mathrm{A} > \mathrm{B} \,.\, \supset : \mathrm{X} \,\varepsilon\, \mathrm{G} \,.\, n\mathrm{X} < \mathrm{A} \,.\, \mathrm{A} - n\mathrm{X} \mathrel{\overset{=}{<}} \mathrm{B} \,.\, -=_{\mathrm{X}} \Lambda$ [$1.3.4.5.6.7.8_1$]

[(α) Hp. P45 : $\supset : \mathrm{Y} \,\varepsilon\, \mathrm{G} \,.\, n\mathrm{Y} \mathrel{\overset{=}{<}} \mathrm{B} -=_{\mathrm{Y}} \Lambda$

(β) » $\mathrm{Y} \,\varepsilon\, \mathrm{G} \,.\, n\mathrm{Y} \mathrel{\overset{=}{<}} \mathrm{B} : \supset : n\mathrm{Y} < \mathrm{A} \,.\,$ P32 : $\supset : m \,\varepsilon\, \mathrm{N} \,.\, (m+1)(n\mathrm{Y}) > \mathrm{A} \mathrel{\overset{=}{>}} m(n\mathrm{Y}) \,.\, -=_{m} \Lambda$

(γ_1) Hp. $\mathrm{Y} \,\varepsilon\, \mathrm{G} . n\mathrm{Y} \mathrel{\overset{=}{<}} \mathrm{B} . m \,\varepsilon\, \mathrm{N} . \mathrm{A} = m(n\mathrm{Y}) : \supset : m \mathrel{\overset{=}{>}} 2$

(γ) » (γ_1) $: \supset : \mathrm{A} - n((m-1)\mathrm{Y}) \mathrel{\overset{=}{<}} \mathrm{B}$

(δ) » $(m+1)(n\mathrm{Y}) > \mathrm{A} > m(n\mathrm{Y}) : \supset : \mathrm{A} - n(m\mathrm{Y}) < \mathrm{B}$

» (α) . (β) . (γ) . (δ) . P1 : $\supset$: Ts]

50. $\mathrm{A} < \mathrm{B} \,.\, \supset : \mathrm{X} \,\varepsilon\, \mathrm{G} \,.\, \mathrm{A} < n\mathrm{X} < \mathrm{B} \,.\, -=_{\mathrm{X}} \Lambda$ [$1 . 3 . 4 . 5 . 6 . 7 . 8_1$]

[(α) Hp. P49 : $\supset : \mathrm{X} \,\varepsilon\, \mathrm{G} \,.\, n\mathrm{X} < \mathrm{B} \,.\, \mathrm{B} - n\mathrm{X} < \mathrm{B} - \mathrm{A} \,.\, -=_{\mathrm{X}} \Lambda$

(β) » $\mathrm{X} \,\varepsilon\, \mathrm{G} \,.\, n\mathrm{X} < \mathrm{B} \,.\, \mathrm{B} - n\mathrm{X} < \mathrm{B} - \mathrm{A} : \supset : \mathrm{A} < n\mathrm{X} < \mathrm{B}$

» (α) . (β) : $\supset$: Ts]

§ 6.

$\mathrm{A}, \mathrm{B} \,\varepsilon\, \mathrm{G} \,.\, a, b \,\varepsilon\, \mathrm{R} \,.\, \supset :$

1. $n \,\varepsilon\, \mathrm{N} \,.\, \supset \,.\, n\left(\frac{1}{n}\mathrm{A}\right) = \mathrm{A}$ [$1 . 1_1 . 8_2$]

[(α) Hp. Pp8_2 : $\supset : \mathrm{B} \,\varepsilon\, \mathrm{G} \,.\, \mathrm{B} = \frac{1}{n}\mathrm{A} \,.\, -=_{\mathrm{B}} \Lambda \,.$

(β_1) » $\mathrm{B} \,\varepsilon\, \mathrm{G} \,.\, \mathrm{B} = \frac{1}{n}\mathrm{A} \,.\,$ Def 1 : $\supset : \mathrm{A} = n\mathrm{B}$

(β_2) » . §5P3 : $\supset : n\left(\frac{1}{n}\mathrm{A}\right) = n\mathrm{B}$

(β) » (β_1) . (β_2) : $\supset : n\left(\frac{1}{n}\mathrm{A}\right) = \mathrm{A}$

Hp. (α) . (β) : $\supset$: Ts]

2. $\mathrm{G} \frown \overline{\mathrm{X}\,\varepsilon}\,(\mathrm{X} < \mathrm{A}) -= \Lambda \,.$ [$1 . 1_1 . 4 . 8_2$]

[(α) Hp. $n \,\varepsilon\, 1 + \mathrm{N} \,.\,$ Pp8_2 : $\supset : \frac{1}{n}\mathrm{A} \,\varepsilon\, \mathrm{G}$

(β) » (α) . §5P22 : $\supset : \frac{1}{n}\mathrm{A} < n\left(\frac{1}{n}\mathrm{A}\right)$. P1 . §3P1 : $\supset : \frac{1}{n}\mathrm{A} < \mathrm{A}$

Hp. (α) . (β) : $\supset$: Ts]

2'. P2 = Pp7

3. $m,\ n\,\varepsilon\,\mathrm{N} \,.\, \supset .\, m\left(\frac{1}{n}\mathrm{A}\right) = \frac{m}{n}\mathrm{A}$ [**1 . 3 . 4 . 8**$_2$]

[(α) Hp. $\mathrm{Pp8_2} : \supset : \mathrm{B}\,\varepsilon\,\mathrm{G} \,.\, \mathrm{B} = \frac{1}{n}\mathrm{A} \,.\, - =_{\mathrm{B}} \Lambda$.

(β_1) » $\mathrm{B}\,\varepsilon\,\mathrm{G} \,.\, \mathrm{B} = \frac{1}{n}\mathrm{A} \,.\, \mathrm{Def\,1} : \supset : \mathrm{A} = n\mathrm{B}$. Hp. §5P1, 3, 8 : $\supset$:

$m\mathrm{A} = n(m\mathrm{B}) \,.\, \mathrm{Def\,1} : \supset : \frac{m}{n}\mathrm{A} = m\mathrm{B}$.

(β_2) Hp. $\mathrm{B}\,\varepsilon\,\mathrm{G} \,.\, \mathrm{B} = \frac{1}{n}\mathrm{A} \,.\, \S5\mathrm{P3} : \supset : m\left(\frac{1}{n}\mathrm{A}\right) = m\mathrm{B}$

(β) » $(\beta_1) \,.\, (\beta) : \supset : m\left(\frac{1}{n}\mathrm{A}\right) = \frac{m}{n}\mathrm{A}$

Hp. (α) . (β) : $\supset$: Ts]

4. $m,\ n\,\varepsilon\,\mathrm{N} \,.\, \supset .\, \frac{1}{n}(m\mathrm{A}) = \frac{m}{n}\mathrm{A}$ [8$_2$]

[(α) Hp. $\mathrm{Pp8_2} \,.\, \S5\mathrm{P1} : \supset : \mathrm{B}\,\varepsilon\,\mathrm{G} \,.\, \mathrm{B} = \frac{1}{n}(m\mathrm{A}) \,.\, - =_{\mathrm{B}} \Lambda$.

(β_1) » $\mathrm{B}\,\varepsilon\,\mathrm{G} \,.\, \mathrm{B} = \frac{1}{n}(m\mathrm{A}) \,.\, \mathrm{Def\,1} : \supset : m\mathrm{A} = n\mathrm{B} \,.\, \mathrm{Def\,1} : \supset : \frac{m}{n}\mathrm{A} = \mathrm{B}$

(β) » $(\beta_1) : \supset : \frac{1}{n}(m\mathrm{A}) = \frac{m}{n}\mathrm{A}$.

Hp. (α) . (β) : $\supset$: Ts]

5. $m,\ n\,\varepsilon\,\mathrm{N} \,.\, \supset .\, \frac{m}{n}\mathrm{A}\,\varepsilon\,\mathrm{G}$ [8$_2$]

[Hp. $\S5\mathrm{P1} \,.\, \mathrm{Pp8_2} : \supset : \frac{1}{n}(m\mathrm{A})\,\varepsilon\,\mathrm{G} \,.\, \mathrm{P4} : \supset : \mathrm{Ts}$]

6. $n\,\varepsilon\,\mathrm{N} \,.\, \mathrm{A} = \mathrm{B} \,.\, \supset .\, \frac{1}{n}\mathrm{A} = \frac{1}{n}\mathrm{B}$. [**1 . 1**$_1$ **. 8**$_2$]

[Hp. $\mathrm{P1} : \supset : \mathrm{A} = n\left(\frac{1}{n}\mathrm{B}\right) \,.\, \mathrm{Def\,1} : \supset : \mathrm{Ts}$]

7. $m,\ n\,\varepsilon\,\mathrm{N} \,.\, \mathrm{A} = \mathrm{B} \,.\, \supset .\, \frac{m}{n}\mathrm{A} = \frac{m}{n}\mathrm{B}$ [**1 . 1**$_1$ **. 8**$_2$]

[Hp. $\S5\mathrm{P3} : \supset : m\mathrm{A} = m\mathrm{B} \,.\, \mathrm{P6} : \supset : \frac{1}{n}(m\mathrm{A}) = \frac{1}{n}(m\mathrm{B}) \,.\, \mathrm{P4} : \supset : \mathrm{Ts}$]

8. $n\,\varepsilon\,\mathrm{N} \,.\, \supset .\, \frac{1}{n}(n\mathrm{A}) = \mathrm{A}$ [**1 . 1**$_1$ **. 8**$_2$]

[(α) Hp. $\mathrm{B}\,\varepsilon\,\mathrm{G} \,.\, \mathrm{B} = n\mathrm{A} \,.\, \mathrm{Def\,1} : \supset : \mathrm{A} = \frac{1}{n}\mathrm{B}$

(β) » $\mathrm{P6} : \supset : \frac{1}{n}\mathrm{B} = \frac{1}{n}(n\mathrm{A})$.

Hp. §5P1 . (α) . (β) : $\supset$: Ts]

9. $m, m', n, n' \varepsilon \mathrm{N} . \frac{m}{n} = \frac{m'}{n'} . \supset . \frac{m}{n}\mathrm{A} = \frac{m'}{n'}\mathrm{A}$. [1 . 3 . 4 . 8_2]

[Hp. $\mathrm{Tr} : \supset : mn' = nm'$. Hp. §5P4, 8 : $\supset : n'(m\mathrm{A}) = n(m'\mathrm{A})$. Def 1 . P3 : $\supset : m\mathrm{A} = n\left(\frac{1}{n'}(m'\mathrm{A})\right)$. P3 . §5P5 . P5 : $\supset : m\mathrm{A} = n\left(\frac{m'}{n'}\mathrm{A}\right)$. Def 1 : $\supset$: Ts]

10. $m, n \,\varepsilon\, \mathrm{N} . a = \frac{m}{n} . \supset . a\mathrm{A} = \frac{m}{n}\mathrm{A}$ [1 . 3 . 4 . 8_2]

[(α) Hp. $\mathrm{Tr} : \supset : m', n' \varepsilon \mathrm{N} . \mathrm{D}(m', n') = 1 . a = \frac{m'}{n'} . - =_{m', n'} \Lambda$.

(β_1) » $m', n' \varepsilon \mathrm{N}. a = \frac{m'}{n'} . \mathrm{Tr} : \supset : \frac{m}{n} = \frac{m'}{n'}$. Hp. P9 : $\supset : \frac{m}{n}\mathrm{A} = \frac{m'}{n'}\mathrm{A}$.

(β_2) » $\mathrm{D}(m', n') = 1$. Def 2 : $\supset : a\mathrm{A} = \frac{m'}{n'}\mathrm{A}$

(β) » » (β_1).(β_2) . P5 : $\supset : a\mathrm{A} = \frac{m}{n}\mathrm{A}$.

Hp. (α) . (β) : $\supset$: Ts]

11. $a\mathrm{A} \,\varepsilon\, \mathrm{G}$ [8_2]

[(α) Hp. $\mathrm{Tr} : \supset : \mathrm{num}\left[\overline{(m, n)} \,\varepsilon\, \left(m, n \,\varepsilon\, \mathrm{N}.\mathrm{D}(m, n) = 1 . a = \frac{m}{n}\right)\right] = 1$

(β) » $m, n \,\varepsilon\, \mathrm{N}.\mathrm{D}(m, n) = 1 . a = \frac{m}{n}$. P5 . Def 2 . I §4P10 : $\supset : a\mathrm{A} \,\varepsilon\, \mathrm{G}$.

Hp. (α) . (β) : $\supset$: Ts]

12. $\mathrm{G} = \mathrm{RG}$ [8_2]

13. $a = b . \supset . a\mathrm{A} = b\mathrm{A}$ [8_2]

[(α) Hp. $\mathrm{Tr} : \supset : m, n \,\varepsilon\, \mathrm{N} . \mathrm{D}(m, n) = 1 . a = b = \frac{m}{n} . - =_{m, n} \Lambda$.

(β) » $m, n \,\varepsilon\, \mathrm{N} . \mathrm{D}(m, n) = 1 . a = b = \frac{m}{n}$. P5 . Def 2 : $\supset : a\mathrm{A} = \frac{m}{n}\mathrm{A}$. $b\mathrm{A} = \frac{m}{n}\mathrm{A} : \supset : a\mathrm{A} = b\mathrm{A}$

Hp. (α) . (β) : $\supset$: Ts]

14. $\mathrm{A} = \mathrm{B} . \supset . a\mathrm{A} = a\mathrm{B}$ [1 . 1_1 . 8_2]

[(α) Hp. $\mathrm{Tr} : \supset : m, n \,\varepsilon\, \mathrm{N} . \mathrm{D}(m, n) = 1 . a = \frac{m}{n} . - =_{m, n} \Lambda$.

(β_1) » $m, n \,\varepsilon\, \mathrm{N}.\mathrm{D}(m, n) = 1 . a = \frac{m}{n}$. Def 2 : $\supset : a\mathrm{A} = \frac{m}{n}\mathrm{A} . a\mathrm{B} = \frac{m}{n}\mathrm{B}$.

(β) » (β_1) . P5, 7 : $\supset : a\mathrm{A} = a\mathrm{B}$.

Hp. (α) . (β) : $\supset$: Ts]

15. $n \varepsilon \mathrm{N} . \supset . \frac{1}{n}(\mathrm{A}+\mathrm{B})=\frac{1}{n}\mathrm{A}+\frac{1}{n}\mathrm{B}$. [1.3.4.$8_2$]

[Hp. P1 : $\supset$: $\mathrm{A}=n\left(\frac{1}{n}\mathrm{A}\right) . \mathrm{B}=n\left(\frac{1}{n}\mathrm{B}\right)$. $\mathrm{Pp}8_2$. §2P3 . §5P6 : $\supset$:

$\mathrm{A}+\mathrm{B}=n\left(\frac{1}{n}\mathrm{A}+\frac{1}{n}\mathrm{B}\right)$. Def1 : $\supset$: Ts]

15'. $m, n \varepsilon \mathrm{N} . \supset . \frac{m}{n}(\mathrm{A}+\mathrm{B})=\frac{m}{n}\mathrm{A}+\frac{m}{n}\mathrm{B}$ [1.3.4.8_2]

[Hp. §5P6 : $\supset$: $m(\mathrm{A}+\mathrm{B})=m\mathrm{A}+m\mathrm{B}$. P15, 4 . §2P3 : $\supset$: Ts]

16. $a(\mathrm{A}+\mathrm{B})=a\mathrm{A}+a\mathrm{B}$ [1.3.4.8_2]

[(α) Hp. Tr : $\supset$: $m, n \varepsilon \mathrm{N} . \mathrm{D}(m, n)=1 . a=\frac{m}{n} . -=_{m, n} \Lambda$.

(β_1) » $m, n \varepsilon \mathrm{N} . \mathrm{D}(m, n)=1 . a=\frac{m}{n}$. Def 2 : $\supset$: $a(\mathrm{A}+\mathrm{B})=$

$\frac{m}{n}(\mathrm{A}+\mathrm{B}) . a\mathrm{A}=\frac{m}{n}\mathrm{A} . a\mathrm{B}=\frac{m}{n}\mathrm{B}$

(β) Hp. $m, n \varepsilon \mathrm{N} . \mathrm{D}(m, n)=1 . a=\frac{m}{n}$ §2P3 . P15', 5 . (β_1) : $\supset$:

$a(\mathrm{A}+\mathrm{B})=a\mathrm{A}+a\mathrm{B}$

Hp. (α) . (β) : $\supset$: Ts]

16'. $n \varepsilon 1+\mathrm{N} . f \varepsilon \mathrm{G}/\mathrm{Z}_n . \supset . a(\Sigma_1{}^n f)=\Sigma_1{}^n a(f)$ [1.3.4.8_2]

17. $(a+b)\mathrm{A}=a\mathrm{A}+b\mathrm{A}$ [1.3.4.8_2]

[(α) Hp. Tr : $\supset$: $m, m', n \varepsilon \mathrm{N} . a=\frac{m}{n}, b=\frac{m'}{n} . -=_{m, m', n} \Lambda$.

(β_1) » $m, m', n \varepsilon \mathrm{N}$. §5P7 . P15 : $\supset$: $\frac{m+m'}{n}\mathrm{A}=\frac{m}{n}\mathrm{A}+\frac{m'}{n}\mathrm{A}$

(β_2) » $a=\frac{m}{n}, b=\frac{m'}{n}$. Tr : $\supset$: $a+b=\frac{m+m'}{n}$

(β) » » (β_1) . (β_2) . P10 : $\supset$: $(a+b)\mathrm{A}$

$=a\mathrm{A}+b\mathrm{A}$.

Hp. (α) . (β) : $\supset$: Ts]

17'. $n \varepsilon 1+\mathrm{N} . f \varepsilon \mathrm{R}/\mathrm{Z}_n . \supset . (\Sigma_1{}^n f)\mathrm{A}=\sum_{r=1}^{r=n}(fr)\mathrm{A}$ [1.3.4.8_2]

18. $a(b\mathrm{A})=(ab)\mathrm{A}$ [1.3.4.8_2]

[(α_1) Hp. $m, m', n, n' \varepsilon \mathrm{N} . \frac{m'}{n'}\mathrm{A}=\mathrm{B}$. Def1 . §5P3, 8 : $\supset$: $(mm')\mathrm{A}=$

$(mn')\mathrm{B}$.P6, 4:$\supset$: $\frac{mm'}{nn'}\mathrm{A}=\frac{mn'}{nn'}\mathrm{B}$. Tr. P9 :$\supset$: $\left(\frac{m}{n}\times\frac{m'}{n'}\right)\mathrm{A}=\frac{m}{n}\mathrm{B}$.

(α) Hp. $m, m', n, n' \varepsilon \mathrm{N} . \frac{m'}{n'}\mathrm{A}=\mathrm{B}$.($\alpha_1$).P14 :$\supset$: $\frac{m}{n}\left(\frac{m'}{n'}\mathrm{A}\right)=\left(\frac{m}{n}\times\frac{m'}{n'}\right)\mathrm{A}$.

(β) Hp. Tr . P5 : $\supset$: $m, m', n, n' \varepsilon$ N . B ε G . $a = \frac{m}{n} . b = \frac{m'}{n'} . \frac{m'}{n'}$A
$=$ B . $-=_{m,\, m',\, n,\, n',\, \mathrm{B}} \Lambda$.
Hp. $(\alpha) . (\beta)$. P10 . Tr : $\supset$: Ts]

19. A $>$ B . $\supset$. aA $>$ aB . [1 . 3 . 4 . 8_2]
[Hp. . $\supset$: A ε B $+$ G . P14, 16, 11 : $\supset$: aA ε aB $+$ G : $\supset$: Ts]

20. $a > b$. $\supset$. aA $>$ bA . [1 . 3 . 4 . 8_2]
[Hp. Tr : $\supset$: $a \,\varepsilon\, b +$ R . P13, 17, 11 : $\supset$: aA ε bA $+$ G : $\supset$: Ts]

21. A $>$ B . $a \stackrel{=}{>} b$. $\supset$. aA $>$ bB [1 . 3 . 4 . 8_2]

22. A $\stackrel{=}{>}$ B . $a > b$. $\supset$. aA $>$ bB [1 . 3 . 4 . 8_2]

23. aA $=$ aB . $\supset$. A $=$ B [1 . 3 . 4 . 5 . 6 . 8_2]
[A, B ε G . $a \,\varepsilon$ R . A $-=$ B . §3P27 . P11, 19 : $\supset$: aA $-=$ aB . I§2P1 .·. $\supset$.·. P23]

24. aA $>$ aB . $\supset$. A $>$ B [1 . 3 . 4 . 5 . 6 . 8_2]

25. aA $=$ bA . $\supset$. $a = b$ [1 . 3 . 4 . 5 . 6 . 8_2]

26. aA $>$ bA . $\supset$. $a > b$ [1 . 3 . 4 . 5 . 6 . 8_2]

27. A $>$ B . $\supset$. a(A $-$ B) $=$ aA $-$ aB [1 . 2 . 3 . 4 . 8_2]
[Hp. §4P5 . P14, 16 : $\supset$: aA $=$ aB $+$ a(A $-$ B) . §4Def1 : $\supset$: Ts]

28. $a > b$. $\supset$. $(a - b)$A $=$ aA $-$ bA [1 . 2 . 3 . 4 . 8_2]
[Hp. Tr : $\supset$: $a = b + (a - b)$. P13, 17 : $\supset$: aA $=$ bA $+$ $(a-b)$A .
§4Def1 : $\supset$: Ts]

29. U ε G . A $<$ B . $\supset$: $x \,\varepsilon$ R . A $<$ xU $<$ B . $-=_x \Lambda$ [1.3.4.5.6.8_1.8_2]
[(α) Hp. §5P33 : $\supset$: $a, b \,\varepsilon$ N . aA $<$ bU $<$ aB . $-=_{a.\,b} \Lambda$.
(β) » $a, b \,\varepsilon$ N . aA $<$ bU $<$ aB . P11, 19 : $\supset$: $\frac{1}{a}(a\mathrm{A}) < \frac{1}{a}(b\mathrm{U})$
$< \frac{1}{a}(a\mathrm{B})$. Tr . P18 . §3P1 : $\supset$: A $<$ $\frac{b}{a}$ U $<$ B .
Hp. $(\alpha) . (\beta)$. Tr . P10 : $\supset$: Ts]

§ 7.

1. A ε KG . B ε A . $\supset$. B $\stackrel{=}{<}$ l'A [1 . 4 . 5]
[(α) Hp. A $\frown$ (B $+$ G) $- =$ Λ . Def1 : $\supset$: B $<$ l'A
(β_1) » A $\frown$ (B $+$ G) $=$ Λ . X ε G . X $<$ B . Def1 : $\supset$: X $<$ l'A
(β_2) » » X $<$ l'A . §3P22, 1, 2 : $\supset$: X $<$ B
(β) » $(\beta_1) . (\beta_2)$. Def2 : $\supset$: B $=$ l'A
Hp. $(\alpha) . (\beta)$. I §3P1'' : $\supset$: Ts]

2. $A \varepsilon KG . \mathrm{l}'A \varepsilon G . \supset : B \varepsilon G . A \frown (B + G) = \Lambda . -=_B \Lambda .$ [1 4 . 5]

3. $A \varepsilon KG . \mathrm{l}'A, B \varepsilon G . A \frown (B + G) = \Lambda . \supset . \mathrm{l}'A \stackrel{=}{<} B$ [5]

[Hp. $B < \mathrm{l}'A . \mathrm{Def}1 : \supset : A \frown (B + G) -= \Lambda .$ Hp. $\mathrm{I}\S 3\mathrm{P}1 : \supset :$ Hp. $B < \mathrm{l}'A . = \Lambda .$
$\mathrm{I}\S 3\mathrm{P}8 . \S 3\mathrm{P}23 : \supset : \mathrm{P}3$]

4. $A \varepsilon KG . X, Y \varepsilon G . X \stackrel{=}{<} Y . Y \stackrel{=}{<} \mathrm{l}'A . \supset . X \stackrel{=}{<} \mathrm{l}'A$ 1 . 4]

5. $A, B \varepsilon G . \supset : m \varepsilon N . mA \stackrel{=}{>} B . -=_m \Lambda$ [1 . 3 . 4 . 5 6 . 8]

[(α) Hp. $NA \frown (B + G) = \Lambda . \mathrm{Pp}8 : \supset : C \varepsilon G . \mathrm{l}'(NA) = C . -=_C \Lambda .$

(β) $A \varepsilon G . \S 5\mathrm{P}22 : \supset : NA \frown (A + G) -= \Lambda . \mathrm{Def}1 : \supset : A < \mathrm{l}'(NA).$

(γ_1) Hp. $C \varepsilon G . C = \mathrm{l}'(NA) . (\beta) . \S 3\mathrm{P}1 : \supset : A < C . \S 4\mathrm{P}28 : \supset : C - A < C.$

(γ) » $(\gamma_1) . \mathrm{P}4 : \supset : C - A < \mathrm{l}'(NA) . \mathrm{Def}1 : \supset m \varepsilon N.$
$C - A < mA -=_m \Lambda .$

(δ) Hp. $C \varepsilon G . m \varepsilon N . C - A < mA . \S 4\mathrm{P}19 . \S 5\mathrm{Def}2 : \supset : C < (m+1)A$

(ε) » $NA \frown (B+G) = \Lambda . (\alpha) . (\beta) . (\gamma) . (\delta) : \supset : \mathrm{l}'(NA) \varepsilon G . H\ NA .$
$H > \mathrm{l}'(NA) . -=_H \Lambda .$

(ε) $\mathrm{P}1 . \S 3\mathrm{P}25 : \supset .\because . A, B \varepsilon G . NA \frown (B+G) = \Lambda : = \Lambda . \mathrm{I}\S 3\mathrm{P}8 .\because . \because . \mathrm{P}5$]

5'. $\mathrm{Pp}8_1 = \mathrm{P}5$

6. $A \varepsilon KG . n \varepsilon N . \mathrm{l}'A \varepsilon G . \supset . \mathrm{l}'(nA) = n(\mathrm{l}'A)$ [1 . 3 . 4 . 5 . 6 7 . 8]

(α) Hp. $\mathrm{P}2 . \mathrm{Pp}8 : \supset : \mathrm{l}'(nA) \varepsilon G .$

(β_1) » $X \varepsilon G . X < \mathrm{l}'(nA) . \mathrm{Def}1 : \supset : A' \varepsilon A . X < nA' . -= \Lambda$

(β_2) » $X \varepsilon G . A' \varepsilon A . X < nA' . \mathrm{P}1 : \supset : X < n(\mathrm{l}'A)$

(β) » $(\alpha) . (\beta_1) . (\beta_2) : \supset : X \varepsilon G . X < \mathrm{l}'(nA) . \supset . X < n(\mathrm{l}'A) .$

(γ_1) » $X \varepsilon G . X < n(\mathrm{l}'A) . \S 5\mathrm{P}50 : \supset : Y \varepsilon G . X < nY < n(\mathrm{l}'A) . -=_Y \Lambda$

(γ_2) » $X, Y \varepsilon G . X < nY < n(\mathrm{l}'A) : \supset : Y < \mathrm{l}'A : \supset : A' \varepsilon A . Y < A' . -=_{A'} \Lambda$

(γ_3) » $A' \varepsilon A . Y < A' : \supset : X < nA' : \supset :$
$X < \mathrm{l}'(nA)$

(γ) Hp. $(\alpha) . (\gamma_1) . (\gamma_2) . (\gamma_3) : \supset : X \varepsilon G . X < \mathrm{l}'(nA) . \supset . X < n(\mathrm{l}'A)$

» $(\beta) . (\gamma) . \mathrm{Def}2 : \supset : \mathrm{Ts}$]

7. $A \varepsilon G . n \varepsilon N . \supset . \mathrm{l}'\left(G \frown \overline{X \varepsilon} (X \varepsilon nG . X \stackrel{=}{<} A)\right) = A$ [1.3.4.5..7.8]

[(α_1) Hp. $Y \varepsilon G . Y < A . \S 5\mathrm{P}50 : \supset : Z \varepsilon G . Y < nZ < A . -=_Z \Lambda$

(α_2) » $Y, Z \varepsilon G . Y < nZ < A : \supset : Y < \mathrm{l}'\left(G \frown \overline{X \varepsilon} (X \varepsilon nG . X \stackrel{=}{<} A)\right)$

(α) » $(\alpha_1) . (\alpha_2) : \supset : Y \varepsilon G . Y < A . \supset . Y < \mathrm{l}'\left(G \frown \overline{X \varepsilon} (X \varepsilon nG . X \stackrel{=}{<} A)\right)$

(β) » $Y \varepsilon G . Y < \mathrm{l}'\left(G \frown \overline{X \varepsilon} (X \varepsilon nG . X \stackrel{=}{<} A)\right) : \supset : Y < A$

» $(\alpha) . (\beta) . \mathrm{Def}2 : \supset : \mathrm{Ts}$]

8. $A \varepsilon G . n \varepsilon N . \supset . \frac{1}{n} A \varepsilon G$ [1 . 3 . 4 . 5 . 6 7 . 8]

$[(\alpha)$ Hp. $P6.Pp8 : \supset : H \,\varepsilon\, G . l'(G \frown \overline{nX\,\varepsilon}\,(nX \stackrel{=}{<} A)) = nH . - =_{H} \Lambda .$

(β) » $H \,\varepsilon\, G . l'(G \frown \overline{nX\,\varepsilon}\,(nX \stackrel{=}{<} A)) = nH . P7 : \supset : nH = A$

» $(\alpha) . (\beta) . \S 6 Def 1 : \supset : Ts]$

8'. $Pp8_2 = P8$

9. $a \,\varepsilon\, KR . l'a \,\varepsilon\, Q . A \,\varepsilon\, G . \supset . l'(aA) \,\varepsilon\, G$ **[1 . 3 . 4 . 5 . 6 . 7 . 8]**

$[(\alpha)$ Hp. $\S 6 P11 : \supset : aA \,\varepsilon\, KG .$

(β) » $Tq : \supset : b \,\varepsilon\, R . a \frown (b + R) = \Lambda . - =_{b} \Lambda .$

(γ) » $b \,\varepsilon\, R . a \frown (b + R) = \Lambda : \supset : B \,\varepsilon\, G . aA \frown (B + G) = \Lambda . - =_{B} \Lambda$

» $(\alpha) . (\beta) . (\gamma) . Pp8 : \supset : Ts]$

10. $A, B \,\varepsilon\, G . a \,\varepsilon\, KR . l'a \,\varepsilon\, Q . A = B . \supset . l'(aA) = l'(aB)$ **[1.3.4.5.6.7.8]**

11. $A \,\varepsilon\, G . a, b \,\varepsilon\, KR . l'a, l'b \,\varepsilon\, Q . l'a = l'b . \supset . l'(aA) = l'(bA)$ [»]

$[(\alpha)$ Hp. $P9 : \supset : l'(aA), l'(bA) \,\varepsilon\, G .$

(β_1) » $X \,\varepsilon\, G . X < l'(aA) . Def 1 : \supset : a' \varepsilon\, a . X < a'A . - =_{a'} \Lambda$

(β_2) » $a' \varepsilon\, a . Tq : \supset : b' \varepsilon\, b . a' < b' . - =_{b'} \Lambda$

(β_3) » $a' \varepsilon\, a . b' \varepsilon\, b . a' < b' . X \,\varepsilon\, G . X < a'A : \supset : X < b'A$

(β_4) » $b' \varepsilon\, b . X \,\varepsilon\, G . X < b'A : \supset : bA \frown (X + G) - = \Lambda : \supset : X < l'(bA)$

(β) » $(\beta_1) . (\beta_2) . (\beta_3) . (\beta_4) : \supset : X \,\varepsilon\, G . X < l'(aA) . \supset . X < l'(bA)$

(γ) » $\begin{pmatrix} a, b \\ b, a \end{pmatrix} \beta : \supset : X \,\varepsilon\, G . X < l'(bA) . \supset . X < l'(aA) .$

» $(\alpha) . (\beta) . (\gamma) : \supset : Ts]$

12. $A \,\varepsilon\, G . a, b \,\varepsilon\, KR . l'a, l'b \,\varepsilon\, Q . \supset . l'(a(bA)) = l'((ab)A)$ **[1.3.4.5.6.7.8]**

13. $A, B \,\varepsilon\, KG . A - = \Lambda . B - = \Lambda . C \,\varepsilon\, G . (A \cup B) \frown (C + G) = \Lambda . \supset . l'(A + B) = l'A + l'B$
[1.3.4.5.6.7.8]

$[(\alpha)$ Hp. $: \supset : A + B \frown (2C + G) = \Lambda .$ Hp. $Pp8 : \supset : l'(A + B), l'A, l'B \,\varepsilon\, G .$

(β_1) » $X \,\varepsilon\, G . X < l'A + l'B . (\alpha) . \S 5 P41 : \supset : X_1, X_2 \,\varepsilon\, G . X_1 + X_2 = X .$
$X_1 < l'A . X_2 < l'B . - =_{X_1, X_2} \Lambda$

(β_2) » $X_1, X_2 \varepsilon\, G . X_1 < l'A . X_2 < l'B : \supset : A' \varepsilon\, A . B' \varepsilon\, B . X_1 < A' . X_2 < B' .$
$- =_{A', B'} \Lambda$

(β_3) » $X, X_1, X_2 \,\varepsilon\, G . A' \varepsilon\, A . B' \varepsilon\, B . X = X_1 + X_2 . X_1 < A' . X_2 < B' : \supset :$
$X < A' + B' : \supset : A + B \frown (X + G) - = \Lambda .$

(β) » $(\beta_1) . (\beta_2) . (\beta_3) : \supset : X \,\varepsilon\, G . X < l'A + l'B . \supset . X < l'(A + B) .$

(γ_1) » $Y \,\varepsilon\, G . Y < l'(A + B) : \supset : A' \varepsilon\, A . B' \varepsilon\, B . Y < A' + B' . - =_{A', B'} \Lambda$

(γ_2) » $A' \varepsilon\, A . B' \varepsilon\, B . P1 : \supset : A' \stackrel{=}{<} l'A . B' \stackrel{=}{<} l'B . (\alpha) : \supset : A' + B' \stackrel{=}{<} l'A + l'B$

(γ_3) » » $Y \,\varepsilon\, G . Y < A' + B' . (\gamma_2) : \supset : Y < l'A + l'B$

(γ) » $(\gamma_1) . (\gamma_2) . (\gamma_3) : \supset : Y \,\varepsilon\, G . Y < l'(A + B) . \supset . Y < l'A + l'B .$

» $(\alpha) . (\beta) . (\gamma) . Def 2 : \supset : Ts]$

14. $A \varepsilon G . a \varepsilon KR . l'a \varepsilon R . \supset . l'(aA) = (l'a)A$ **[1.3.4.5.6.7.8]**

[(α) Hp. P9 . §6P11 $: \supset : l'(aA), (l'a)A \varepsilon G$.

(β_1) » $X \varepsilon G . X < (l'a)A . (\alpha)$. §6P29 $: \supset : x \varepsilon R . X < xA < (l'a)A . - =_x \Lambda$

(β_2) » $X \varepsilon G . x \varepsilon R . xA < (l'a)A : \supset : x < l'a$. Tq $: \supset : a' \varepsilon a . x < a' . - =_a \Lambda$

(β_3) » $X \varepsilon G . a' \varepsilon a . x \varepsilon R . X < xA . x < a' : \supset : X < a'A : \supset : aA \frown (X + G) - = \Lambda$

(β) » $(\beta_1) . (\beta_2) . (\beta_3) : \supset : X \varepsilon G . X < (l'a)A . \supset . X < l'(aA)$

(γ_1) » $Y \varepsilon G . Y < l'(aA) : \supset : a' \varepsilon a . Y < a'A . - =_{a'} \Lambda$

(γ_2) » $a' \varepsilon a$. Tq $: \supset : a' \leqq l'a$. Hp. $: \supset : a'A \leqq (l'a)A$

(γ) » $(\gamma_1) . (\gamma_2) : \supset : Y \varepsilon G . Y < l'(aA) . \supset . Y < (l'a)A$

» $(\alpha) . (\beta) . (\gamma)$. Def 2 $: \supset :$ Ts]

§ 8.

$A, B \varepsilon G . m, n \varepsilon Q . \supset :$

1. $mA \varepsilon G$. **[1.3.4.5.6.7.8]**

[Hp. Def 1, 2 $: \supset : mA = l' \{ (R \frown \overline{x \varepsilon} (x \leqq m)) A \}$. §7P9 . Tq . I§4P10 $: \supset :$ Ts]

2. $QG = G$ **[1.3.4.5.6.7.8]**

3. $a \varepsilon KR . l'a = m . \supset . mA = (l'a)A$ [»]

[Hp. Tq $: \supset : l'a = l'(R \frown \overline{x \varepsilon} (x \leqq m))$. §7P11 . Def 1, 2 $: \supset :$ Ts]

4. $A = B . \supset . nA = nB$ **[1.3.4.5.6.7.8]**

5. $m = n . \supset . mA = nA$ [»]

6. $A = B . m = n . \supset . mA = nB$ [»]

7. $m(A + B) = mA + mB$ [»]

[(α) Hp. Tq $: \supset : a \varepsilon KR . l'a = m . - =_a \Lambda$

(β) » $a \varepsilon KR \, l'a \varepsilon Q$. §6P11 $: \supset : aA, aB \varepsilon KG$

(γ) » $a \varepsilon KR . l'a \varepsilon Q . (\beta)$. §7P13 $: \supset : l'(aA + aB) = l'(aA) + l'(aB)$

» $(\alpha) . (\gamma)$. P3 . §7P9 . Def 1, 2 $: \supset :$ Ts]

7'. $a \varepsilon N . f \varepsilon G|Z_a . \supset . m(\Sigma_1^a f) = \Sigma_1^a (mf)$. **[1.3.4.5.6.7.8]**

8. $(m + n)A = mA + nA$ [»]

8'. $a \varepsilon N . f \varepsilon Q|Z_a . \supset . (\Sigma_1^a f)A = \Sigma_{r=1}^{r=a} (fr)A$ [»]

9. $m(nA) = (mn)A$ [»]

10. $A > B . \supset . mA > mB$ [»]

11. $m > n . \supset . mA > nA$ [»]

12. $A > B . m \geqq n . \supset . mA > nB$ [»]

13. $A \geqq B . m > n . \supset . mA > nB$ [»]

14. $mA = mB \,.\, \supset .\, A = B$ $[1.3.4.5.6.7.8]$
15. $mA > mB \,.\, \supset .\, A > B$ [»]
16. $mA = nA \,.\, \supset .\, m = n$ [»]
17. $mA > nA \,.\, \supset .\, m > n$ [»]
18. $A > B \,.\, \supset .\, m(A - B) = mA - mB$ [»]
19. $m > n \,.\, \supset .\, (m - n)A = mA - nA$ [»]

§ 9.

$A, B, C, D, U, U' \varepsilon G \,.\, \supset :$

1. $l'(A/U) -= \Lambda$ $[1.3.4.5.8_1]$

$[(\alpha_1)$ Tr $: \supset : x \varepsilon R \,.\, n, nx \varepsilon N \,.\, nx \stackrel{=}{<} n \,.\, -=_{n,x} \Lambda$

(α_2) Hp. $x \varepsilon R \,.\, n, nx \varepsilon N \,.\, nx \stackrel{=}{<} n \,.\, U \stackrel{=}{<} A$. §5 P5, 11, 12, 13 $: \supset :$ $(nx)U \stackrel{=}{<} nA$

(α) » $U \stackrel{=}{<} A \,.\, (\alpha_1) \,.\, (\alpha_2) : \supset : x \varepsilon R . n, nx \varepsilon N . (nx)U \stackrel{=}{<} nA . -=_{n,x} \Lambda$

(β_1) » $U > A$. $Pp8_1 : \supset : n \varepsilon N \,.\, U \stackrel{=}{<} nA \,.\, -=_n \Lambda$

(β_2) » $n \varepsilon N$. Tr $: \supset : x \varepsilon R \,.\, nx = 1 \,.\, -=_x \Lambda$

(β_3) » $x \varepsilon R \,.\, n \varepsilon N \,.\, U \stackrel{=}{<} nA \,.\, nx = 1$. §5 P5, 11 $: \supset : (nx)U \stackrel{=}{<} nA$

(β) » $U > A . (\beta_1) . (\beta_2) . (\beta_3) : \supset : x \varepsilon R . n, nx \varepsilon N . (nx)U \stackrel{=}{<} nA . -=_{n,x} \Lambda$

» $(\alpha) \,.\, (\beta)$. Pp5 $: \supset :$ Ts$]$

2. $A/U \,\varepsilon\, Q$ $[1.3.4.5.6.8_1]$

$[(\alpha)$ Hp. Def1, 2 . P1 $: \supset : x \varepsilon l'(A/U) \,.\, n, nx \varepsilon N \,.\, (nx)U \stackrel{=}{<} nA \,.\, -=_{n,x} \Lambda$

(β) » $n \varepsilon N$. §5P31 $: \supset : m \varepsilon N \,.\, nA \stackrel{=}{<} mU \,.\, -=_m \Lambda$

(γ) » $x \varepsilon R \,.\, m, n, nx \varepsilon N \,.\, (nx)U \stackrel{=}{<} nA \stackrel{=}{<} mU$. §3P1, 2 $: \supset : (nx)U$ $\stackrel{=}{<} mU$. §5P16, 17 $: \supset : nx \stackrel{=}{<} m$. Tr $: \supset : x \stackrel{=}{<} \frac{m}{n}$.

(δ) » $(\alpha) . (\beta) . (\gamma)$. Tr $: \supset \therefore h \varepsilon R \,.\, l'(A/U) \frown (h + R) = \Lambda : -=_h \Lambda$

» (δ) . Tq $: \supset :$ Ts$]$

3. $\bar{l}'(A/U) = R \frown \overline{x \varepsilon}\, (xU \stackrel{=}{<} A)$ $[1.3.4.5.6.8_2]$

$[(\alpha_1)$ Hp. $x \varepsilon R . x \varepsilon \bar{l}'(A/U)$. Def1, 2 $: \supset : n, nx \varepsilon N . (nx)U \stackrel{=}{<} nA . -=_n \Lambda$

(α_2) » $x \varepsilon R . n, nx \varepsilon N . (nx)U \stackrel{=}{<} nA$. §6P18, 23, 24 $: \supset : xU \stackrel{=}{<} A$

(α) » $(\alpha_1) . (\alpha_2) : \supset : x \varepsilon R . x \varepsilon \bar{l}'(A/U) \,.\, \supset .\, x \varepsilon \left(R \frown \overline{x \varepsilon}\, (xU \stackrel{=}{<} A)\right)$

(β_1) » $x \varepsilon \left(R \frown \overline{x \varepsilon}\, (xU \stackrel{=}{<} A)\right) . n, nx \varepsilon N$. §5P3, 9 . §6P18 $: \supset : (nx)U \stackrel{=}{<} A$

(β_2) » $x \varepsilon R$: Tr $: \supset : n, nx \varepsilon N \,.\, -=_n \Lambda$

(β) » $(\beta_1) \,.\, (\beta_2) : \supset : x \varepsilon \left(R \frown \overline{x \varepsilon}\, (xU \stackrel{=}{<} A)\right) \,.\, \supset .\, x \varepsilon \bar{l}'(A/U)$

» $(\alpha) \,.\, (\beta)$. I §4P2 $: \supset :$ Ts$]$

4 — *Formul.*

4. $(\mathrm{A}/\mathrm{U})\mathrm{U} = \mathrm{A}$ [1 . 3 . 4 . 5 . 6 . 7 . 8]

[(α) Hp. P2 . §8P1 $: \supset : (\mathrm{A}/\mathrm{U})\mathrm{U} \,\varepsilon\, \mathrm{G}$.

(β_1) » $(\alpha) . \mathrm{X} \,\varepsilon\, \mathrm{G} . \mathrm{X} < (\mathrm{A}/\mathrm{U})\mathrm{U}$. Def1, 2 . P3 $: \supset : x \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U})$.
$\mathrm{X} < x\mathrm{U} . -\!=_x \Lambda$

(β_2) » $\mathrm{X} \,\varepsilon\, \mathrm{G} . x \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}) . \mathrm{X} < x\mathrm{U}$. P3 $: \supset : \mathrm{X} < \mathrm{A}$

(β) » $(\beta_1) . (\beta_2) : \supset : \mathrm{X} \,\varepsilon\, \mathrm{G} . \mathrm{X} < (\mathrm{A}/\mathrm{U})\mathrm{U} . \supset . \mathrm{X} < \mathrm{A}$

(γ_1) » $\mathrm{X} \,\varepsilon\, \mathrm{G} . \mathrm{X} < \mathrm{A}$. §6P29 $: \supset : x \,\varepsilon\, \mathrm{R} . \mathrm{X} < x\mathrm{U} < \mathrm{A} . -\!=_x \Lambda$

(γ_2) » $\mathrm{X} \,\varepsilon\, \mathrm{G} . x \,\varepsilon\, \mathrm{R} . \mathrm{X} < x\mathrm{U} < \mathrm{A} : \supset : \mathrm{X} < \mathrm{l}'\{(\overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}))\mathrm{U}\}$

(γ) » $(\gamma_1) . (\gamma_2) : \supset : \mathrm{X} \,\varepsilon\, \mathrm{G} . \mathrm{X} < \mathrm{A} . \supset . \mathrm{X} < (\mathrm{A}/\mathrm{U})\mathrm{U}$

» $(\alpha) . (\beta) . (\gamma)$. Def1, 2 . §8Def2 $: \supset :$ Ts]

4'. $\mathrm{G} = \mathrm{QU}$ [1 . 3 . 4 . 5 . 6 . 7 . 8]

5. $\mathrm{A} = \mathrm{B} . \supset . \mathrm{A}/\mathrm{U} = \mathrm{B}/\mathrm{U}$ [1 . 3 . 4 . 5 . 6 . 8_1]

[Hp. §5P3.§3P1.Def1, 2.I §4P2$:\supset:\overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U})=\overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U})$. P1, 2 . Tq $:\supset:$ Ts]

6. $\mathrm{U} = \mathrm{U}' . \supset . \mathrm{A}/\mathrm{U} = \mathrm{A}/\mathrm{U}'$ [1 . 3 . 4 . 5 . 6 . 8_1]

7. $\mathrm{A} > \mathrm{B} . \supset . \mathrm{A}/\mathrm{U} > \mathrm{B}/\mathrm{U}$ [»]

[(α) Hp. §5P33 $: \supset : m, n \,\varepsilon\, \mathrm{N} . m\mathrm{A} > n\mathrm{U} > m\mathrm{B} . -\!=_{m\,n} \Lambda$

(β) » $m, n \,\varepsilon\, \mathrm{N} . m\mathrm{A} > n\mathrm{U} > m\mathrm{B}$. §5P4 $: \supset : m\mathrm{A} > \left(m\frac{n}{m}\right)\mathrm{U} > m\mathrm{B}$

(γ) » P1.(α).$(\beta)$$:\supset.\because. h \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}).\overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U}) \frown (h\text{+}\mathrm{R} \cup \iota h) = \Lambda : -\!=_h \Lambda$

» P2 . Tq $: \supset :$ Ts]

8. $\mathrm{U} > \mathrm{U}' . \supset . \mathrm{A}/\mathrm{U} < \mathrm{A}/\mathrm{U}'$ [1 . 3 . 4 . 5 . 6 . 8_1]

9. $\mathrm{A}/\mathrm{U} = \mathrm{B}/\mathrm{U} . \supset . \mathrm{A} = \mathrm{B}$ [»]

[P7.§3P21.P2.Tq $:\supset:$ $\mathrm{A}, \mathrm{B}, \mathrm{U} \,\varepsilon\, \mathrm{G}.\mathrm{A}-\!=\mathrm{B}.\supset.\mathrm{A}/\mathrm{U}-\!=\mathrm{B}/\mathrm{U}$.I §2P1 $:\supset:$ P9]

10. $\mathrm{A}/\mathrm{U} = \mathrm{A}/\mathrm{U}' . \supset . \mathrm{U} = \mathrm{U}'$ [1 . 3 . 4 . 5 . 6 . 8_1]

11. $\mathrm{A}/\mathrm{U} > \mathrm{B}/\mathrm{U} . \supset . \mathrm{A} > \mathrm{B}$ [»]

12. $\mathrm{A}/\mathrm{U} > \mathrm{A}/\mathrm{U}' . \supset . \mathrm{U} < \mathrm{U}'$ [»]

13. $(\mathrm{A} + \mathrm{B})/\mathrm{U} = \mathrm{A}/\mathrm{U} + \mathrm{B}/\mathrm{U}$ [1 . 3 . 4 . 5 . 6 . 7 . 8_1]

[(α_1) Hp. $x \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}) . y \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U})$. Def 1, 2 . P1 $: \supset : n, nx, ny \,\varepsilon\, \mathrm{N}$.
$(nx)\mathrm{U} \stackrel{=}{<} n\mathrm{A} . (ny)\mathrm{U} \stackrel{=}{<} n\mathrm{B} . -\!=_n \Lambda$

(α_2) » $x, y \,\varepsilon\, \mathrm{R} . n, nx, ny \,\varepsilon\, \mathrm{N} . (nx)\mathrm{U} \stackrel{=}{<} n\mathrm{A} . (ny)\mathrm{U} \stackrel{=}{<} n\mathrm{B} : \supset :$
$(n(x + y))\mathrm{U} \stackrel{=}{<} n(\mathrm{A} + \mathrm{B})$

(α) » $(\alpha_1) . (\alpha_2) : \supset : \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}) + \overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U}) \supset \mathrm{l}'((\mathrm{A} + \mathrm{B})/\mathrm{U})$

(β_1) » $z \,\varepsilon\, \mathrm{l}'((\mathrm{A} + \mathrm{B})\mathrm{U})$. Def1, 2 . P1 $: \supset : n, nz \,\varepsilon\, \mathrm{N} . (nz)\mathrm{U} \stackrel{=}{<}$
$n(\mathrm{A} + \mathrm{B}) . -\!=_n \Lambda$

(β_2) Hp. $z \,\varepsilon\, \mathrm{R} \,.\, n, nz \,\varepsilon\, \mathrm{N} \,.\, (nz)\mathrm{U} \overline{<} n(\mathrm{A}+\mathrm{B}) \,.\, \S 5\mathrm{P}47 : \supset : x, y \,\varepsilon\, \mathrm{R} \,.$
$m, mx, my \,\varepsilon\, \mathrm{N} \,.\, x+y=z \,.\, (mx)\mathrm{U} \overline{<} m\mathrm{A} \,.\, (my)\mathrm{U} \overline{<}$
$m\mathrm{B} \,.\, -=_{m,x,y} \Lambda$

(β) » $(\beta_1) . (\beta_2) : \supset : \overline{\mathrm{l}'}((\mathrm{A}+\mathrm{B})/\mathrm{U}) \supset \mathrm{l}'(\mathrm{A}/\mathrm{U}) + \overline{\mathrm{l}'}(\mathrm{B}/\mathrm{U})$

» Def 2 . I §4P2 . $(\alpha) . (\beta) : \supset : \overline{\mathrm{l}'}((\mathrm{A}+\mathrm{B})/\mathrm{U}) = \overline{\mathrm{l}'}(\mathrm{A}/\mathrm{U}) +$
$\mathrm{l}'(\mathrm{B}/\mathrm{U})$. P2 . Def 1, 2 . Tq : ɔ : Ts]

13'. $n \,\varepsilon\, \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{G}/\mathrm{Z}_n \,.\, \supset \,.\, (\Sigma_1{}^n f)/\mathrm{U} = \Sigma_1{}^n(f/\mathrm{U})$ [1 . 3 . 4 . 5 . 6 . 7 . 8_1]

14. $\mathrm{A} > \mathrm{B} \,.\, \supset \,.\, (\mathrm{A}-\mathrm{B})/\mathrm{U} = \mathrm{A}/\mathrm{U} - \mathrm{B}/\mathrm{U}$ [»]

[Hp. $:\supset: \mathrm{A}=\mathrm{B}+(\mathrm{A}-\mathrm{B})$.P5, 13$:\supset: \mathrm{A}/\mathrm{U}=\mathrm{B}/\mathrm{U}+(\mathrm{A}-\mathrm{B})\mathrm{U}$.P2.Tq:ɔ:Ts]

15. $a \,\varepsilon\, \mathrm{N} \,.\, \supset \,.\, (a\mathrm{A})/\mathrm{U} = a(\mathrm{A}/\mathrm{U})$ [1 . 3 . 4 . 5 . 6 . 8_1]

[(α_1) Hp. $x \,\varepsilon\, \mathrm{l}'((a\mathrm{A})/\mathrm{U})$.Def 1, 2.P1$:\supset: n, nx \,\varepsilon\, \mathrm{N}.(nx)\mathrm{U} \overline{<} n(a\mathrm{A}).-=_n \Lambda$

(α_2) » $x \,\varepsilon\, \mathrm{R} \,.\, n, nx \,\varepsilon\, \mathrm{N} \,.\, (nx)\mathrm{U} \overline{<} n(a\mathrm{A})$. §5P20, 25 . Tr : ɔ : $m,$
$m\dfrac{x}{a} \,\varepsilon\, \mathrm{N} \,.\, \left(m\dfrac{x}{a}\right)\mathrm{U} \overline{<} m\mathrm{A} \,.\, -=_m \Lambda$

(α) » $(\alpha_1) . (\alpha_2) : \supset : \mathrm{l}'((a\mathrm{A})/\mathrm{U}) \supset a(\overline{\mathrm{l}'}(\mathrm{A}/\mathrm{U}))$

(β_1) » $x \,\varepsilon\, a\,\overline{\mathrm{l}'}(\mathrm{A}/\mathrm{U})$. Def 1, 2 : ɔ : $n, n\dfrac{x}{a} \,\varepsilon\, \mathrm{N} \,.\, \left(n\dfrac{x}{a}\right)\mathrm{U} \overline{<} n\mathrm{A}. -=_n \Lambda$

(β_2) » $x \,\varepsilon\, \mathrm{R} \,.\, n, n\dfrac{x}{a} \,\varepsilon\, \mathrm{N} \,.\, \left(n\dfrac{x}{a}\right)\mathrm{U} \overline{<} n\mathrm{A} \,.\, \supset \,.\, (nx)\mathrm{U} \overline{<} n(a\mathrm{A})$

(β) » $(\beta_1) . (\beta_2) : \supset : a(\overline{\mathrm{l}'}(\mathrm{A}/\mathrm{U})) \supset \mathrm{l}'((a\mathrm{A})/\mathrm{U})$

» $(\alpha) . (\beta)$. P2 . Tq : ɔ : Ts]

16. $a \,\varepsilon\, \mathrm{R} \,.\, \supset \,.\, (a\mathrm{A})/\mathrm{U} = a(\mathrm{A}/\mathrm{U})$ [1 . 3 . 4 . 5 . 6 . 8_1 . 8_2]

17. $a \,\varepsilon\, \mathrm{Q} \,.\, \supset \,.\, (a\mathrm{A})/\mathrm{U} = a(\mathrm{A}/\mathrm{U})$ [1 . 3 . 4 . 5 . 6 . 7 . 8]

[Hp. §8P1 : ɔ : $a\mathrm{A} \,\varepsilon\, \mathrm{G}$. P4 : ɔ : $((a\mathrm{A})/\mathrm{U})\mathrm{U} = a\mathrm{A}$. P4 : ɔ : $((a\mathrm{A})/\mathrm{U})\mathrm{U}$
$= a((\mathrm{A}/\mathrm{U})\mathrm{U})$. P2 . §8P9 : ɔ : $((a\mathrm{A})/\mathrm{U})\mathrm{U} = (a(\mathrm{A}/\mathrm{U}))\mathrm{U}$. §8
P16 : ɔ : Ts]

18. $a \,\varepsilon\, \mathrm{N} \,.\, \supset \,.\, (a\mathrm{U})/\mathrm{U} = a$ [1 . 3 . 4 . 5 . 6]

[(α) Hp. Def 1, 2 : ɔ : $a \,\varepsilon\, \overline{\mathrm{l}'}((a\mathrm{U})/\mathrm{U})$

(β_1) » $x \,\varepsilon\, \mathrm{l}'((a\mathrm{U})/\mathrm{U})$.Def 1, 2.P1$:\supset: n, nx \,\varepsilon\, \mathrm{N}.(nx)\mathrm{U} \overline{<} n(a\mathrm{U}).-=_n \Lambda$

(β_2) » $x \,\varepsilon\, \mathrm{R}.n, nx \,\varepsilon\, \mathrm{N}.(nx)\mathrm{U} < n(a\mathrm{U})$. §5P8, 16, 17 . Tr :ɔ: $x < a$

(β) » $(\beta_1) . (\beta_2) : \supset : x \,\varepsilon\, \overline{\mathrm{l}'}((a\mathrm{U})/\mathrm{U}) \,.\, \supset \,.\, x \overline{<} a$

» $(\alpha) . (\beta)$. Def 1, 2 . Tq : ɔ : Ts]

19. $\mathrm{A}/\mathrm{A} = 1$ [1 . 3 . 4 . 5 . 6 . 8_1]

[Hp. §5 Def 1 : ɔ : $1\mathrm{A} = \mathrm{A}$. P5 : ɔ : $(1\mathrm{A})/\mathrm{A} = \mathrm{A}/\mathrm{A}$. P18 : ɔ : Ts]

20. $a \,\varepsilon\, \mathrm{R} \,.\, \supset \,.\, (a\mathrm{U})/\mathrm{U} = a$ [1 . 3 . 4 . 5 . 6 . 8_1 . 8_2]

[Hp. P16 : ɔ : $(a\mathrm{U})/\mathrm{U} = a(\mathrm{U}/\mathrm{U})$. P19 . Tr : ɔ : Ts]

21. $a \varepsilon Q . \supset . (aU)/U = a$ **[1 . 3 . 4 . 5 . 6 . 7 . 8]**

[Hp. P17 : ɔ : $(aU)/U = a(U/U)$. P19 . Tq : ɔ : Ts]

21′. $A = B . = . A/B = 1$ **[1 . 3 . 4 . 5 . 6 . 8_1]**

[(α) Hp. $A = B$. P5, 19 : ɔ : $A/B = 1$

(β) » $A/B = 1$. P19 : ɔ : $A/B = B/B$. P9 : ɔ : $A = B$

» (α) . (β) : ɔ : Ts]

21″. $A > B . = . A/B > 1$ **[1 . 3 . 4 . 5 . 6 . 8_1]**

21‴. $A < B . = . A/B < 1$ [»]

22. $a \varepsilon N . \supset : A/U = a . = . A = aU$ [»]

[(α) Hp. $A/U = a$. P18 : ɔ : $A/U = (aU)/U$. P9 : ɔ : $A = aU$

(β) » $A = aU$. P5, 18 : ɔ : $A/U = a$

(α) . (β) : ɔ : Ts]

23. $a \varepsilon R . \supset : A/U = a . = . A = aU$ **[1 . 3 . 4 . 5 . 6 . 8_1 . 8_2]**

24. $a \varepsilon Q . \supset : A/U = a . = . A = aU$ **[1 . 3 . 4 . 5 . 6 . 7 . 8]**

25. $a \varepsilon N . \supset : A/U > a . = . A > aU$ **[1 . 3 . 4 . 5 . 6 . 8_1]**

[(α) Hp. $A/U > a$. P18 : ɔ : $A/U > (aU)/U$. P11 : ɔ : $A > aU$

(β) » $A > aU$. P7 : ɔ : $A/U > (aU)/U$. P18 : ɔ : $A/U > a$

(α) . (β) : ɔ : Ts]

26. $a \varepsilon R . \supset : A/U > a . = . A > aU$ **[1 . 3 . 4 . 5 . 6 . 8_1 . 8_2]**

27. $a \varepsilon Q . \supset : A/U > a . = . A > aU$ **[1 . 3 . 4 . 5 . 6 . 7 . 8]**

31. $a \varepsilon N . \supset . (aA)/(aU) = A/U$ **[1 . 3 . 4 . 5 . 6 . 8_1]**

[(α_1) Hp. $x \varepsilon \bar{\Gamma}((aA)/(aU))$. Def1, 2 . P1 : ɔ : n, $nx \varepsilon N . (nx)(aU) \overline{\overline{<}}$ $n(aA) . -=_x \Lambda$

(α_2) » $x \varepsilon R . n, nx \varepsilon N . (nx)(aU) \overline{\overline{<}} n(aA)$. §5 P8, 14, 15 : ɔ : $(nx)U \overline{\overline{<}} nA$

(α) » (α_1) . (α_2) : ɔ : $\bar{\Gamma}((aA)/(aU)) \supset \bar{\Gamma}(A/U)$

(β_1) » $x \varepsilon \bar{\Gamma}(A/U)$. Def1, 2 . P1 :ɔ: $n, nx \varepsilon N.(nx)U \overline{\overline{<}} nA -=_x \Lambda$

(β_2) » $x \varepsilon R . n, nx \varepsilon N . (nx)U \overline{\overline{<}} nA$. §5 P3, 9, 8 : ɔ : $(nx)(aU)$ $\overline{\overline{<}} n(aA)$

(β) » (β_1) . (β_2) : ɔ : $\bar{\Gamma}(A/U) \supset \bar{\Gamma}((aA)/(aU))$

» (α) . (β) : ɔ : $\bar{\Gamma}((aA)/(aU)) = \bar{\Gamma}(A/U)$. Def1, 2 . P2 . Tq:ɔ:Ts]

32. $a \varepsilon R . \supset . (aA)/(aU) = A/U$ **[1 . 3 . 4 . 5 . 6 . 8_1 . 8_2]**

33. $a \varepsilon Q . \supset . (aA)/(aU) = A/U$ **[1 . 3 . 4 . 5 . 6 . 7 . 8]**

34. $a \varepsilon N . \supset . A/(aU) = \frac{1}{a}(A/U)$ **[1 . 3 . 4 . 5 . 6 . 8_1]**

[Hp. P31, 15 $: \supset : a\big(\mathrm{A}/(a\mathrm{U})\big) = \mathrm{A}/\mathrm{U} \,.\, \mathrm{P2} \,.\, \mathrm{Tq} : \supset : \mathrm{Ts}$]

35. $a \,\varepsilon\, \mathrm{R} \,.\supset.\, \mathrm{A}/(a\mathrm{U}) = \frac{1}{a}(\mathrm{A}/\mathrm{U})$ [1 . 3 . 4 . 5 . 6 . 8_1 . 8_2]

36. $a \,\varepsilon\, \mathrm{Q} \,.\supset.\, \mathrm{A}/(a\mathrm{U}) = \frac{1}{a}(\mathrm{A}/\mathrm{U})$ [1 . 3 . 4 . 5 . 6 . 7 . 8]

37. $(\mathrm{A}/\mathrm{B})(\mathrm{B}/\mathrm{U}) = \mathrm{A}/\mathrm{U}$ [1 . 3 . 4 . 5 . 6 . 8_1]

[(α_1) Hp. $a \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{B}) \,.\, b \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U}) \,.\, \mathrm{Def1, 2} \,.\, \mathrm{P1} \,.\, \mathrm{Tr} : \supset : n, na, nb \,\varepsilon\, \mathrm{N}.$
$(na)\mathrm{B} \overline{\overline{<}}\, n\mathrm{A} \,.\, (nb)\mathrm{U} \,\overline{\overline{<}}\, n\mathrm{B} \,.\, -=_n \Lambda$

(α_2) » $a, b \,\varepsilon\, \mathrm{R} \,.\, n, na, nb \,\varepsilon\, \mathrm{N} \,.\, (na)\mathrm{B} \,\overline{\overline{<}}\, n\mathrm{A} \,.\, (nb)\mathrm{U}$
$\overline{\overline{<}}\, n\mathrm{B} \,.\, \S 5\mathrm{P3}, 9, 8 \,.\, \S 3\mathrm{P1}, 2 \,.\, \mathrm{Tr} : \supset : (n^2ab)\mathrm{U} \,\overline{\overline{<}}\, n^2\mathrm{A}$

(α) » $(\alpha_1) \,.\, (\alpha_2) : \supset : \{\overline{\mathrm{l}}'(\mathrm{A}/\mathrm{B})\} \times \{\overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U})\} \supset \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U})$

(β_1) » $c \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}) \,.\, \mathrm{Def1, 2} : \supset : n, nc \,\varepsilon\, \mathrm{N} \,.\, (nc)\mathrm{U} \,\overline{<}\, n\mathrm{A} \,.\, -=_n \Lambda$

(β_2) » $c \,\varepsilon\, \mathrm{R} \,.\, n, nc \,\varepsilon\, \mathrm{N} \,.\, (nc)\mathrm{U} < \mathrm{A} \,.\, \S 5\mathrm{P33} :\supset: n', m \,\varepsilon\, \mathrm{N}.(n'nc)\mathrm{U}$
$< m\mathrm{B} < (n'n)\mathrm{A} \,.\, -=_{n', m} \Lambda$

(β_3) » $c \,\varepsilon\, \mathrm{R} \,.\, n, nc, n', m \,\varepsilon\, \mathrm{N} \,.\, (n'nc)\mathrm{U} < m\mathrm{B} < (n'n)\mathrm{A} \,.\, \mathrm{Tn}.\mathrm{Tr}.$
$\S 5\mathrm{P4} \,.\, \S 3\mathrm{P1} : \supset : \left(m\frac{n'nc}{m}\right)\mathrm{U} < m\mathrm{B} \,.\, \left(n'n\frac{m}{n'n}\right)\mathrm{B} < (n'n)\mathrm{A}$

(β_4) » $(\beta_1) \,.\, (\beta_2) \,.\, (\beta_3) \,.\, \mathrm{Tr} : \supset : c \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}) \,.\supset.\, y, z \,\varepsilon\, \mathrm{R} \,.\, yz = c \,.$
$y \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{B}) \,.\, z \,\varepsilon\, \overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U}) \,.\, -=_{x, y} \Lambda$

(β) » $(\beta_4) : \supset : \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}) \supset \{\overline{\mathrm{l}}'(\mathrm{A}/\mathrm{B})\} \times \{\overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U})\}$

» $(\alpha) \,.\, (\beta) : \supset : \{\overline{\mathrm{l}}'(\mathrm{A}/\mathrm{B})\} \times \{\overline{\mathrm{l}}'(\mathrm{B}/\mathrm{U})\} = \overline{\mathrm{l}}'(\mathrm{A}/\mathrm{U}) \,.\, \mathrm{Def1, 2} \,.\, \mathrm{P2}\,.$
$\mathrm{Tq} : \supset : \mathrm{Ts}$]

37′. $\mathrm{A}/\mathrm{B} = (\mathrm{A}/\mathrm{U})/(\mathrm{B}/\mathrm{U})$ [1 . 3 . 4 . 5 . 6 . 8_1]

37″. $n \,\varepsilon\, 2 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{G}/\mathrm{Z}_n \,.\supset.\, \Pi_{r=1}^{r=n-1} \{(fr)/(f(r+1))\} = (f1)/(fn)$
[1 . 3 . 4 . 5 . 6 . 8_1]

38. $\mathrm{A}/\mathrm{U} = 1/(\mathrm{U}/\mathrm{A})$ [»]

[Hp. P31 $: \supset : (\mathrm{A}/\mathrm{U})(\mathrm{U}/\mathrm{A}) = \mathrm{A}/\mathrm{A} \,.\, \mathrm{P19} : \supset : (\mathrm{A}/\mathrm{U})(\mathrm{U}/\mathrm{A}) = 1 \,.\, \mathrm{P2}\,.$
$\mathrm{Tq} : \supset : \mathrm{Ts}$]

39. $\mathrm{A}/\mathrm{C} = \mathrm{B}/\mathrm{U} \,.=.\, \mathrm{A}/\mathrm{U} = (\mathrm{B}/\mathrm{U})(\mathrm{C}/\mathrm{U})$ [1 . 3 . 4 . 5 . 6 . 8_1]

40. $\mathrm{A}/\mathrm{B} = \mathrm{C}/\mathrm{D} \,.=.\, (\mathrm{A}/\mathrm{U})/(\mathrm{B}/\mathrm{U}) = (\mathrm{C}/\mathrm{U})/(\mathrm{D}/\mathrm{U})$ [»]

41. » $.\supset.\, \mathrm{A}/\mathrm{C} = \mathrm{B}/\mathrm{D}$ [»]

42. » $.\supset.\, \mathrm{D}/\mathrm{B} = \mathrm{C}/\mathrm{A}$ [»]

43. » $.\supset.\, \mathrm{B}/\mathrm{A} = \mathrm{D}/\mathrm{C}$ [»]

44. » $.\supset.\, (\mathrm{A}+\mathrm{B})/\mathrm{B} = (\mathrm{C}+\mathrm{D})/\mathrm{D}$ [1 . 3 . 4 . 5 . 6 . 7 . 8_1]

[Hp. P2 . Tq . P19 $: \supset : \mathrm{A}/\mathrm{B} + \mathrm{B}/\mathrm{B} = \mathrm{C}/\mathrm{D} + \mathrm{D}/\mathrm{D} \,.\, \mathrm{P13} : \supset : \mathrm{Ts}$]

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45. $A/B = C/D \,.\, A > B \,.\supset.\, (A - B)/B = (C - D)/D$ [1.3.4.5.6.7.8_1]

46. » $.\supset.\, (A + B)/(A - B) = (C + D)/(C - D)$
[1.3.4.5.6.7.8_1]

47. $H \varepsilon KG \,.\, U \varepsilon H \,.\, H/U \varepsilon KR \,.\supset:\, U' \varepsilon H \,.\supset_{U'}.\, H/U' \varepsilon KR$ [1.3.4.5.6.8_1]
[Hp. $B,\ U' \varepsilon H$. P31 . Tq $:\supset:$ $B/U' = (B/U) \times \big(1/(U'/U)\big)$.Hp.Tr$:\supset:$Ts]

§ 10.

$V,\ V',\ V_1,\ V_2,\ V_3 \varepsilon KG \,.\supset:$

1. $\omega \varepsilon (\text{pd}.VV') .\cdot. = .\cdot. \omega \varepsilon (V'/V)\,\text{sim} : A, B \varepsilon V.\supset_{A,B}.A/B = (\omega A)/(\omega B)$ (Def)
2. $\omega \varepsilon (\text{pi}.VV') .\cdot. = .\cdot. \omega \varepsilon (V'/V)\,\text{sim} : A, B \varepsilon V.\supset_{A,B}.A/B = (\omega B)/(\omega A)$ (Def)
3. $\omega \varepsilon (\text{pd}.VV') \,.\, A, B \varepsilon V \,.\supset:\, A = B \,.=.\, \omega A = \omega B$
[Hp. P1 . I §5P25 $:\supset:$ Ts]
4. $\omega \varepsilon (\text{pi}.VV') \,.\, A, B \varepsilon V \,.\supset:\, A = B \,.=.\, \omega A = \omega B$
5. $\omega \varepsilon (\text{pd}.VV') \,.\supset.\, \bar\omega \varepsilon (\text{pd}.V'V)$ [1.3.4.5.6.8_1]
[(α) Hp. P1 . I §5P22 $:\supset:$ $\bar\omega \varepsilon (V/V')\,\text{sim}$
(β) » $A, B \varepsilon V$. P1 . I §5P23 $:\supset:$ $\bar\omega(\omega A) = A \,.\, \bar\omega(\omega B) = B$. §9P5,
6 . P1 $:\supset:$ $(\omega A)/(\omega B) = \big(\bar\omega(\omega A)\big)/\big(\bar\omega(\omega B)\big)$
» (α) . (β) . P1 $:\supset:$ Ts]
6. $\omega \varepsilon (\text{pi}.VV') \,.\supset.\, \bar\omega \varepsilon (\text{pi}.V'V)$ [1.3.4.5.6.8_1]
7. $\omega \varepsilon (\text{pd}.VV') \,.\, A, B \varepsilon V \,.\supset:\, A > B \,.=.\, \omega A > \omega B$ [»]
[(α) Hp. $A > B$.§9P21″.P1.Tq $:\supset:$ $(\omega A)/(\omega B) > 1$.§9P21″$:\supset:$ $\omega A > \omega B$
(β) » $\omega A > \omega B$. (α) . P5 $:\supset:$ $\bar\omega(\omega A) > \bar\omega(\omega B)$. I §5P23 $:\supset:$ $A > B$
» (α) . (β) $:\supset:$ Ts]
8. $\omega \varepsilon (\text{pi}.VV').A, B \varepsilon V \,.\supset:\, A > B \,.=.\, \omega A < \omega B$ [1.3.4.5.6.8_1]
9. $\omega \varepsilon (\text{pd}.VV').a \varepsilon N.A, aA \varepsilon V.\supset.\omega(aA) = a(\omega A)$ [»]
[Hp. P1 $:\supset:$ $(aA)/A = \big(\omega(aA)\big)/(\omega A)$. §9P18 $:\supset:$ $a = \big(\omega(aA)\big)/(\omega A)$.
§9P22 $:\supset:$ Ts]

9′. $\omega \varepsilon (\text{pd}.VV') \,.\, a \varepsilon R \,.\, A, aA \varepsilon V \,.\supset.\, \omega(aA) = a(\omega A)$ [1.3.4.5.6.8_1.8_2]

9″. » $a \varepsilon Q$ » [1.3.4.5.6.7.8]

10. $\omega \varepsilon (\text{pi}.VV').a \varepsilon N.A, aA \varepsilon V \,.\supset.\, \omega(aA) = \dfrac{1}{a}(\omega A)$ [1.3.4.5.6.7.8_1]

10′. » $.a \varepsilon R.$ » [1.3.4.5.6.8_1.8_2]

10″. » $.a \varepsilon Q.$ » [1.3.4.5.6.7.8]

11. $\omega \varepsilon (\text{pd}.VV').A, B, A + B \varepsilon V.\supset.\omega(A+B) = \omega A + \omega B$ [1.3.4.5.6.7.8_1]

[(α) Hp. P1 . §9P44 : $\supset$: $(A + B)/B = (\varpi A + \varpi B)/(\varpi B)$
(β) » P1 : $\supset$: $(A + B)/B = (\varpi'A + B))/(\varpi B)$
» (α) . (β) . §9P2 . Tq : $\supset$: $(\varpi(A+B))/(\varpi B) = (\varpi A + \varpi B)/(\varpi B)$.
§9P9 : $\supset$: Ts]

12. $\varpi \varepsilon$ (pd. VV) . A, B, A—B ε V . $\supset$. $\varpi(A—B) = \varpi A — \varpi B$ **[1.3.4.5.6.7.8$_1$]**

13. $\varpi \varepsilon$ (p. VV′) . = : $\varpi \varepsilon$ (pd. VV′) . $\cup$. $\varpi \varepsilon$ (pi. VV′) (Def)

14. $n \varepsilon 2 + N$. $f \varepsilon (KG)|Z_n$. A $\varepsilon f1$: $r \varepsilon Z_{n-1}$. $\supset_r$. $\varpi_r \varepsilon (p.frf(r+1))$: $\supset$.·. $\varpi_{n-1}\varpi_{n-2} \ldots \varpi_2\varpi_1 A = \varpi_{n-1}(\varpi_{n-2} \ldots \varpi_2\varpi_1 A)$ (Def)

15. $n \varepsilon 2 + N$. $f \varepsilon (KG)|Z_{n-1}$. $r \varepsilon Z_{n-1}$. $\supset_r$. $\varpi_r \varepsilon (p . frf(r+1))$: $\supset$.·. $\varpi_{n-1} \varpi_{n-2} \ldots \varpi_2\varpi_1 \varepsilon (fn|f1)$ sim

16. $\varpi_1 \varepsilon (pd . V_1V_2)$. $\varpi_2 \varepsilon (pd . V_2V_3)$. $\supset$. $\varpi_2\varpi_1 \varepsilon (pd . V_1V_3)$ **[1.3.4.5.6.8]**
[(α) Hp. P15 : $\supset$: $\varpi_2\varpi_1 \varepsilon (V_3|V_1)$ sim
(β) » A, B εV_1 . P1 . §9P2 . Tq : $\supset$: $A/B = (\varpi_1 A)|(\varpi_1 B) = (\varpi_2(\varpi_1 A))|(\varpi_2(\varpi_1 B))$. P14 : $\supset$: $A/B = (\varpi_2\varpi_1 A)|(\varpi_2\varpi_1 B)$
» (α) . (β) . P1 : $\supset$: Ts]

17. $\varpi_1 \varepsilon (pd . V_1V_2)$. $\varpi_2 \varepsilon (pi . V_2V_3)$. $\supset$. $\varpi_2\varpi_1 \varepsilon (pi . V_1V_3)$ **[1.3.4.5.6.8$_1$]**

18. $\varpi_1 \varepsilon (pi . V_1V_2)$. » . $\supset$. $\varpi_2\varpi_1 \varepsilon (pd . V_1V_3)$ [»]

19. » . $\varpi_2 \varepsilon (pd . V_2V_3)$. $\supset$. $\varpi_2\varpi_1 \varepsilon (pi . V_1V_3)$ [»]

20. $\varpi_1 \varepsilon (p . V_1V_2)$. $\varpi_2 \varepsilon (p . V_2V_3)$. $\supset$: $\varpi_2\varpi_1 \varepsilon (pd.V_1V_3)$.=. $(\varpi_1 \varepsilon (pd.V_1V_2)$. $\varpi_2 \varepsilon (pd . V_2V_3)) \cup (\varpi_1 \varepsilon (pi . V_1V_2)$. $\varpi_2 \varepsilon (pi .V_2V_3))$ **[1.3.4.5.6.8$_1$]**

21. $\varpi_1 \varepsilon (p . V_1V_2)$. $\varpi_2 \varepsilon (p . V_2V_3)$. $\supset$: $\varpi_1\varpi_2 \varepsilon (pi . V_1V_3)$.=. $(\varpi_1 \varepsilon (pd.V_1V_2)$. $\varpi_2 \varepsilon (pi . V_2V_3)) \cup (\varpi_1 \varepsilon (pi . V_1V_2)$. $\varpi_2 \varepsilon (pd .V_2V_3))$ **[1.3.4.5.6.8$_1$]**

22. $n \varepsilon 2 + N$. $f \varepsilon (KG)|Z_n$: $r \varepsilon Z_{n-1}$. $\supset_r$. $\varpi_r \varepsilon (p . fr(fr+1)$: $\supset$.·. $\varpi_{n-1} \ldots \varpi_2\varpi_1 \varepsilon (pd.f1fn)$.=. num $(\varpi \frown (pi . ff)) \varepsilon (2N \cup \iota 0)$ **[1.3.4.5.6.8$_1$]**

23. $n \varepsilon 2 + N$. $f \varepsilon (KG)|Z_n$: $r \varepsilon Z_{n-1}$. $\supset_r$. $\varpi_r \varepsilon (p . frf(r+1))$.$\supset$.·. $\varpi_{n-1} \ldots \varpi_2\varpi_1 \varepsilon (pi . f1fn)$. = . num $(\varpi \frown (pi . ff)) \varepsilon 2N-1$ **[1.3.4.5.6.8$_1$]**

24. $\varpi \varepsilon (V'|V)$ sim : X, Y ε V . $\supset_{X, Y}$. X+Y ε V . $\varpi(X+Y) = \varpi X + \varpi Y$: $\supset$.·.
24$_1$. A, B ε V . $\supset$: A = B . = . $\varpi A = \varpi B$
24$_2$. A′, B′ ε V . $\supset$: $\bar\varpi(A' + B') = \bar\varpi A' + \bar\varpi B'$ **[1 . 1$_1$]**
[(α) Hp. A, B ε V : $\supset$: $\varpi(A + B) = \varpi A + \varpi B$. P24$_1$. I §5P23: $\supset$: $A + B = \bar\varpi(\varpi A + \varpi B)$. I §5P23 . §2P3 : $\supset$: $\bar\varpi(\varpi A + \varpi B) = \bar\varpi(\varpi A) + \bar\varpi(\varpi B)$
(β) Hp. I §5P23 : $\supset$: A, B ε V . $\varpi A = A'$. $\varpi B = B'$. $- =_{A, B} \Lambda$
» (α) . (β) . §2P3 . P24$_1$: $\supset$: Ts]

24_3. $A, B \varepsilon V . \supset : A > B . = . \varpi A > \varpi B$ **[1 . 1_1]**

[(α) Hp. $A > B : \supset : A \varepsilon B + G$. Hp. $: \supset : \varpi A \varepsilon \varpi B + G : \supset : \varpi A > \varpi B$

(β) » $\varpi A > \varpi B$. $P24_2$. I §5P23 : $\supset$: $A > B$

» (α) . (β) : $\supset$: Ts]

24_4. $A \varepsilon V . m \varepsilon N . \supset . \varpi(mA) = m(\varpi A)$ **[1 . 1_1]**

[(α) Hp. §5P1 : $\supset$: $mA \varepsilon V$

(β) » $P24_1 : \supset : \varpi(1A) = 1(\varpi A) : \supset : 1 \varepsilon \overline{m \varepsilon}$ (Ts)

(γ) » $n \varepsilon \overline{m \varepsilon}$ (Ts) . Pp1 : $\supset$: $\varpi(nA) + \varpi A = n(\varpi A) + \varpi A$.

Hp. $P24_1 : \supset : \varpi\big((n+1)A\big) = (n+1)(\varpi A) : \supset : n+1 \varepsilon \overline{m \varepsilon}$ (Ts)

Hp. (α) . (β) . (γ) . Pi : $\supset$: Ts]

24_5. $A \varepsilon V . a \varepsilon R . \supset . \varpi(aA) = a(\varpi A)$ **[1 . 3 . 4 . 5 . 6 . 8_2]**

[(α) Hp. §6P11 : $\supset$: $aA \varepsilon V$

(β) » Tr : $\supset$: $n, na \varepsilon N . - =_n A$

(γ) » $n, na \varepsilon N . P24_4 : \supset : \varpi\big((na)A\big) = (na)(\varpi A)$. (α).$P24_4$

$: \supset : n\big(\varpi(aA)\big) = (na)(\varpi A)$. §5P8, 14 : $\supset$: $\varpi(aA) = a(\varpi A)$

Hp. (α) . (β) . (γ) : $\supset$: Ts]

24_6. $A \varepsilon V . a \varepsilon KR . l'a \varepsilon Q . \supset . l'\varpi\big((aA)\big) = \varpi\big((l'a)A\big)$ **[1.3.4.5.6.7.8]**

[(α) Hp. §6P11 . §7P9 : $\supset$: $aA \supset V . (l'a)A \varepsilon V$

(β_1) » $X \varepsilon V' . X < l'\big(\varpi(aA)\big) : \supset : a' \varepsilon a . X < \varpi(a'A) . - =_{a'A}$

(β_2) » $X \varepsilon V' . a' \varepsilon a . X < \varpi(a'A)$. Tq : $\supset$: $X < \varpi\big((l'a)A\big)$

(β) » (α).(β_1).(β_2) $: \supset : X \varepsilon V' . X < l'\big(\varpi(aA)\big) . \supset . X < \varpi\big((l'a)A\big)$

(γ_1) » $X \varepsilon V' . X < \varpi\big((l'a)A\big)$. $P24_2$, 24_3 : $\supset$: $\bar{\varpi}X < (l'a)A : \supset :$

$a' \varepsilon a . \bar{\varpi}X < a'A . - =_{a'A}$

(γ_2) » $X \varepsilon V' . a' \varepsilon a . \bar{\varpi}X < a'A : \supset : X < \varpi(a'A) : \supset : X < l'\big(\varpi(aA)\big)$

(γ) » (α).(γ_1).(γ_2)$: \supset : X \varepsilon V' . X < \varpi\big((l'a)A\big) . \supset . X < l'\big(\varpi(aA)\big)$

» (β) . (γ) . §7Def2 : $\supset$: Ts]

24_7. $A \varepsilon V . a \varepsilon KR . l'a \varepsilon Q . \supset . \varpi\big((l'a)A\big) = (l'a)(\varpi A)$ **[1.3.4.5.6.7.8]**

[Hp. $: \supset : \varpi(aA) = a(\varpi A) : \supset : l'\big(\varpi(aA)\big) = l'\big(a(\varpi A)\big)$. $P24_6$ $: \supset :$ Ts]

24_8. $A \varepsilon V . m \varepsilon Q . \supset . \varpi(mA) = m(\varpi A)$ **[1 . 3 . 4 . 5 . 6 . 7 . 8]**

[Hp. §8P3 . $P24_7$: $\supset$: Ts]

24_9. $A, B \varepsilon V . \supset . A/B = (\varpi A)/(\varpi B)$ [»]

[Hp. §9P4 : $\supset$: $A = (A/B)B$. §9P2 . $P24_4$, 24_8 . §9P24 : $\supset$: Ts]

24_{10}. $\varpi \varepsilon$ (pd . VV′) **[1 . 3 . 4 . 5 . 6 . 7 . 8]**

[Hp. $P25_9$. P1 : $\supset$: Ts]

26. $\omega \varepsilon (V'/V)$ sim $.\therefore. m \varepsilon N . X \varepsilon V . \supset_{X, m} : mX \varepsilon V . V/X \varepsilon KR . \omega(mX) = m(\omega X) .\therefore. \supset ::$

26_1. $A, B \varepsilon V . \supset . A = B . = . \omega A = \omega B$

26_2. $A' \varepsilon V' . m \varepsilon N . \supset . \overline{\omega}(mA') = m(\overline{\omega}A')$ [$1 . 1_1$]

26_3. $a \varepsilon R . A, aA \varepsilon V . \supset . \omega(aA) = a(\omega A)$ [$1 . 3 . 4 . 5 . 6 . 8_2$]

26_4. $A, B \varepsilon V . \supset . A/B = (\omega A)/(\omega B)$ [$1 . 3 . 4 . 5 . 6 . 8_1 . 8_2$]

[Hp. §9P23$:\supset:$A$=$(A/B)B.P$26_3$$:\supset:$Ts]

26_5. $A, B \varepsilon V . \supset : A > B . = . \omega A > \omega B$ [»]

[(α) Hp. A$>$B.§9P21''$:\supset:$ A/B$>$1.P26_4.Tr.§9P21''$:\supset:$ ωA$>$$\omega$B

(β) » $\omega A > \omega B . (\alpha) . P26_2 : \supset : A > B$

» $(\alpha) . (\beta) : \supset :$ Ts]

26_6. $\omega \varepsilon$ (pd . VV') [$1 . 3 . 4 . 5 . 6 . 8_1 . 8_2$]

27. $\omega \varepsilon (V'/V)$ sim $.\therefore. m \varepsilon N . X \varepsilon V . \supset_{X, m} : mX \varepsilon V . V/X \varepsilon KR . \omega(mX) = \frac{1}{m}(\omega X) .\therefore. \supset :: \omega \varepsilon$ (pi . VV') [$1 . 3 . 4 . 5 . 6 . 8_1 . 8_2$]

28. $\omega \varepsilon (V'/V)$ sim $.\therefore. m \varepsilon N . X \varepsilon V . Y \varepsilon KV : \supset_{m, X, Y} : mX \varepsilon V . \omega(mX) = m(\omega X) . \bar{1}'(\omega Y) = \omega(1'Y) .\therefore. \supset ::$

28_1. $A, B \varepsilon V . \supset : A = B . = . \omega A = \omega B$

28_2. $A' \varepsilon V . m \varepsilon N . \supset . \overline{\omega}(mA') = m(\overline{\omega}A')$ [$1 . 1_1$]

28_3. $a \varepsilon R . A, aA \varepsilon V . \supset . \omega(aA) = a(\omega A)$ [$1 . 3 . 4 . 5 . 6 . 8_2$]

28_4. $A \varepsilon V . a \varepsilon KR . 1'a \varepsilon Q . aV \supset V . \supset . \omega\big((1'a)A\big) = (1'a)(\omega A)$

[(α) Hp. $P28_3 : \supset : \omega(aA) = a(\omega A) : \supset : 1'\big(\omega(aA)\big) = (1'a)(\omega A)$

(β) » $: \supset : 1'\big(\omega(aA)\big) = \omega\big((1'a)A\big)$

» $(\alpha) . (\beta) : \supset :$ Ts]

28_5. $m \varepsilon Q . A, mA \varepsilon V . \supset . \omega(mA) = m(\omega A)$ [$1 . 3 . 4 . 5 . 6 . 7 . 8$]

29_6. $A, B \varepsilon V . \supset . A/B = (\omega A)/(\omega B)$ »

28_7. $A, B \varepsilon V . \supset : A > B . = . \omega A = \omega B$ »

28_8. $\omega \varepsilon$ (pd . VV') »

29. $\omega \varepsilon (V'/V)$ sim $.\therefore. m \varepsilon N . X \varepsilon V . Y \varepsilon KV : \supset_{m, X, Y} : mX \varepsilon V . \omega(mX) = \frac{1}{m}(\omega X) . 1'(\omega Y) = \omega(1'Y) .\therefore. \supset :: \omega \varepsilon$ (pi . VV') [1.3.4.5.6.7.8]

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