V.

§ 1. — num.

$u, v \, \varepsilon \, \mathrm{K} \, . \supset :$

1. $\mathrm{num}\, u = 0 \, . = . \, u = \Lambda \, .$ Def.
2. $m \, \varepsilon \, \mathrm{N} . \supset :: \mathrm{num}\, u = m . = .\!\cdot\!. \, u - = \Lambda : x \, \varepsilon \, u . \supset_x . \, \mathrm{num}(u - \iota x) = m - 1 \, .$ Def.
3. $\mathrm{num}\, u = 1 \, . = .\!\cdot\!. \, u - = \Lambda : x, y \, \varepsilon \, u \, . \supset_{x,\,y} . \, x = y \, .$
4. $\mathrm{num}\, u = \infty \, . = . \, \mathrm{num}\, u - \varepsilon \, \mathrm{N}_0 \, .$ Def.
5. $\mathrm{num}\, u \, \varepsilon \, \mathrm{N} \cup \iota 0 \cup \iota \infty \, .$
6. $a \, \varepsilon \, \mathrm{N}_0 \, . \supset . \, a + \infty = \infty + a = \infty + \infty = \infty \, . \, a < \infty \, .$ Def.
7. $u \cap v = \Lambda \, . \supset . \, \mathrm{num}\,(u \cup v) = \mathrm{num}\, u + \mathrm{num}\, v \, .$
8. $\mathrm{num}\,(u \cup v) + \mathrm{num}\,(u \cap v) = \mathrm{num}\, u + \mathrm{num}\, v \, .$
9. $k \, \varepsilon \, \mathrm{KK} \, . \supset . \cap ' k = \overline{x \, \varepsilon} \, (y \, \varepsilon \, k \, . \supset_y . \, x \, \varepsilon \, y) \, .$ Def.
10. » $. \supset . \cup ' k = \overline{x \, \varepsilon} \, (y \, \varepsilon \, k \, . \, x \, \varepsilon \, y \, . - =_y \Lambda) \, .$ Def.
11. $u \, \varepsilon \, \mathrm{KK} \, . \, p, q \, \varepsilon \, \mathrm{N} \, . \, \mathrm{num}\, u = p : x \, \varepsilon \, u \, . \supset_x . \, \mathrm{num}\, x = q : x, y \, \varepsilon \, u \, . \, x -$
    $= y \, . \supset_{x,\,y} . \, x \cap y = \Lambda \, .\!\cdot\!. \supset . \, \mathrm{num} \cup ' u = p \times q \, .$
12. $f \, \varepsilon \, (v \, \mathrm{f} \, u) \, . \supset . \, \mathrm{num}\, f u \leq \mathrm{num}\, u \, .$
13. » $\mathrm{num}\, f u = \infty \, . \supset . \, \mathrm{num}\, u = \infty \, .$
14. $f \, \varepsilon \, (v \, \mathrm{f} \, u) \, \mathrm{Sim} \, . \supset . \, \mathrm{num}\, f u = \mathrm{num}\, u \, .$
15. $f \, \varepsilon \, (v \, \mathrm{f} \, u) \, \mathrm{sim} \, . \supset . \, \mathrm{num}\, v = \mathrm{num}\, u \, .$

§ 2. — max, min.

$u, \; v \, \varepsilon \, \mathrm{Kq} \, . \supset :$

1. $x = \max u \, . = . \, x \, \varepsilon \, u \, . \, u \cap (x + \mathrm{Q}) = \Lambda \, .$ Def.
2. $x = \min u \, . = . \, x \, \varepsilon \, u \, . \, u \cap (x - \mathrm{Q}) = \Lambda \, .$ Def.
3. $\mathrm{num}\, u \, \varepsilon \, \mathrm{N} \, . \supset . \max u, \; \min u \, \varepsilon \, \mathrm{q} \, .$
4. $u \, \varepsilon \, \mathrm{KN} \, . \, u - = \Lambda \, . \supset . \min u \, \varepsilon \, \mathrm{N} \, .$
5. $u \, \varepsilon \, \mathrm{KN} \, . \, u - = \Lambda \, . \, m \, \varepsilon \, \mathrm{N} \, . \, u \cap (m + \mathrm{N}) = \Lambda \, . \supset . \max u \, \varepsilon \, \mathrm{N} \, .$
6. $u \, \varepsilon \, \mathrm{Kn} \, . \, u - = \Lambda \, . \, m \, \varepsilon \, \mathrm{n} \, . \, u \cap (m + \mathrm{N}) = \Lambda \, . \supset . \max u \, \varepsilon \, \mathrm{n} \, .$
7. » » » $. \, u \cap (m - \mathrm{N}) = \Lambda \, . \supset . \min u \, \varepsilon \, \mathrm{n} \, .$
8. $\min \mathrm{N} = 1 \, . \max \mathrm{N} = \Lambda \, .$
9. $\max \mathrm{Q} = \Lambda \, . \min \mathrm{Q} = \Lambda \, . \max \mathrm{q} = \Lambda \, . \min \mathrm{q} = \Lambda \, .$
10. $\max u, \; \max v \, \varepsilon \, \mathrm{q} \, . \supset . \max (u \cup v) = \max (\max u, \; \max v) \, .$
11. $\min u, \; \min v \, \varepsilon \, \mathrm{q} \, . \supset . \min (u \cup v) = \min (\min u, \; \min v) \, .$
12. $\max u, \; \max v \, \varepsilon \, \mathrm{q} \, . \supset . \max (u + v) = \max u + \max v \, .$

13. $\min u,\ \min v \varepsilon \mathrm{q} . \supset . \min(u+v) = \min u + \min v$.
14. $\max u \varepsilon \mathrm{q} . \supset . \min(-u) = -\max u$.
15. $\min u \varepsilon \mathrm{q} . \supset . \max(-u) = -\min u$.
16. $u,\ v \varepsilon \mathrm{KQ} . \max u,\ \max v \varepsilon \mathrm{Q} . \supset . \max(u \times v) = \max u \times \max v$.

§ 3. — l′, l_1.

$u,\ v \varepsilon \mathrm{Kq} . u -= \Lambda . v -= \Lambda . \supset :$

1. $x \varepsilon \mathrm{q} . \supset :: x = \mathrm{l}'u . = .\cdot . u \cap (x+\mathrm{Q}) = \Lambda : y \varepsilon x - \mathrm{Q} . \supset_y . u \cap (y+\mathrm{Q}) -= \Lambda$. Def.

1′. $x \varepsilon \mathrm{q} . \supset :: x = \mathrm{l}_1 u . = .\cdot . u \cap (x-\mathrm{Q}) = \Lambda : y \varepsilon x + \mathrm{Q} . \supset_y . u \cap (y-\mathrm{Q}) -= \Lambda$. Def.

2. $\max u \varepsilon \mathrm{q} . \supset . \max u = \mathrm{l}'u$.
2′. $\min u \varepsilon \mathrm{q} . \supset . \min u = \mathrm{l}_1 u$.
3. $\mathrm{l}'u \varepsilon u . \supset . \mathrm{l}'u = \max u$.
3′. $\mathrm{l}_1 u \varepsilon u . \supset . \mathrm{l}_1 u = \min u$.
4. $m \varepsilon \mathrm{q} . u \cap (m+\mathrm{Q}) = \Lambda . \supset . \mathrm{l}'u \varepsilon \mathrm{q} . \mathrm{l}'u \leqq m$.
4′. $m \varepsilon \mathrm{q} . u \cap (m-\mathrm{Q}) = \Lambda . \supset . \mathrm{l}_1 u \varepsilon \mathrm{q} . \mathrm{l}_1 u \geqq m$.
5. $\mathrm{l}'u = \infty . = : m \varepsilon \mathrm{q} . \supset_m . u \cap (m+\mathrm{Q}) -= \Lambda$. Def.
5′. $\mathrm{l}_1 u = -\infty . = : m \varepsilon \mathrm{q} . \supset_m . u \cap (m-\mathrm{Q}) -= \Lambda$. Def.
6. $\mathrm{l}'u \varepsilon \mathrm{q} \cup \iota\infty$.
6′. $\mathrm{l}_1 u \varepsilon \mathrm{q} \cup \iota(-\infty)$.
7. $a \varepsilon \mathrm{q} . \supset . a+\infty = \infty + a = \infty . a - \infty = (-\infty) + a = -\infty . \infty + \infty = \infty . -\infty - \infty = -\infty . -\infty < a < +\infty . -\infty < +\infty$. Def.
8. $a \varepsilon \mathrm{Q} . \supset . a \times \infty = \infty \times a = \infty . a \times (-\infty) = (-\infty) \times a = -\infty . \infty \times \infty = \infty . (-\infty) \times (-\infty) = \infty . \infty \times (-\infty) = (-\infty) \times \infty = -\infty . a/\infty = a/(-\infty) = 0 . a/0 = \pm\infty$. Def.
9. $\mathrm{l}'(u \cup v) = \max(\mathrm{l}'u,\ \mathrm{l}'v)$.
9′. $\mathrm{l}_1(u \cup v) = \min(\mathrm{l}_1 u,\ \mathrm{l}_1 v)$.
10. $u \supset v . \supset . \mathrm{l}'u \leqq \mathrm{l}'v . \mathrm{l}_1 u \geqq \mathrm{l}_1 v$.

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§ 3. 1-6. WEIERSTRASS. V. PINCHERLE, *Saggio di una introduzione alla teoria delle funzioni analitiche secondo i principii del prof. Weierstrass.* Giornale di Battaglini, XVIII, p. 242.

BOLZANO (1817). V. STOLZ, *Vorlesungen über Allgemeine Arithmetik*, I, p. 149.

DINI. *Fondamenti per la teorica delle funzioni di variabili reali.* Pisa, 1878, N. 15.

11. $u \supset v : x \varepsilon v . \supset_x . u \cap (x + \mathrm{Q}) -= \Lambda .\cdot. \supset . \mathrm{l}'u = \mathrm{l}'v .$
11'. $u \supset v : x \varepsilon v . \supset_x . u \cap (x - \mathrm{Q}) -= \Lambda .\cdot. \supset . \mathrm{l}_1 u = \mathrm{l}_1 v .$
12. $\mathrm{l}_1 u \leq \mathrm{l}'u .$
13. $\mathrm{num}\, u > 1 . \supset . \mathrm{l}_1 u < \mathrm{l}'u .$
14. $\mathrm{l}'(u + v) = \mathrm{l}'u + \mathrm{l}'v . \mathrm{l}_1(u + v) = \mathrm{l}_1 u + \mathrm{l}_1 v .$
15. $m \varepsilon \mathrm{Q} . \supset . \mathrm{l}'(mu) = m\mathrm{l}'u . \mathrm{l}_1(mu) = m\mathrm{l}_1 u .$
16. $\mathrm{l}'(-u) = -\mathrm{l}_1 u . \mathrm{l}_1(-u) = -\mathrm{l}'u .$
17. $u, v \varepsilon \mathrm{KQ} . \supset . \mathrm{l}'(u \times v) = \mathrm{l}'u \times \mathrm{l}'v .$
17'. $u, v \varepsilon \mathrm{KQ} . \supset . \mathrm{l}_1(u \times v) = \mathrm{l}_1 u \times \mathrm{l}_1 v .$
18. $u \varepsilon \mathrm{KQ} . \supset . \mathrm{l}'(/u) = /\mathrm{l}_1 u . \mathrm{l}_1(/u) = /\mathrm{l}'u .$
19. $\mathrm{l}'\mathrm{Q} = \infty . \mathrm{l}_1\mathrm{Q} = 0 . \mathrm{l}'\mathrm{q} = \infty . \mathrm{l}_1\mathrm{q} = -\infty .$
20. $u \varepsilon \mathrm{KQ} . \supset .\cdot. \mathrm{l}_1 u = 0 . = : h \varepsilon \mathrm{Q} . \supset_h . u \cap (h - \mathrm{Q}) -= \Lambda .$
21. » . » $= : h \varepsilon \mathrm{Q} . \supset_h . \mathrm{num}\, [u \cap (h - \mathrm{Q})] = \infty$
22. $u , v \varepsilon \mathrm{KQ} . \supset : \mathrm{l}_1 (u \cup v) = 0 . = . \mathrm{l}_1 u = 0 . \cup . \mathrm{l}_1 v = 0 .$
23. $u , v \varepsilon \mathrm{Kq} . \supset : \mathrm{l}' (u \cup v) = \infty . = . \mathrm{l}' u = \infty . \cup . \mathrm{l}' v = \infty .$

§ 4. — q_n.

1. $n \varepsilon \mathrm{N} . \supset : \mathrm{q}_n = \mathrm{q}\, \mathrm{f}\, \mathrm{Z}_n .$
2. $x \varepsilon \mathrm{q}_n . \supset . x = (x_1 , x_2 , \ldots x_n) .$
3. $x, y \varepsilon \mathrm{q}_n . \supset : x = y . = . x_1 = y_1 . x_2 = y_2 \ldots x_n = y_n .$
4. $x, y \varepsilon \mathrm{q}_n . \supset . x + y = (x_1 + y_1 , \ldots x_n + y_n) .$
5. » » $x - y = (x_1 - y_1 , \ldots x_n - y_n) .$ Def.
6. $a \varepsilon \mathrm{q} . x \varepsilon \mathrm{q}_n . \supset . ax = (ax_1 , ax_2 , \ldots ax_n) .$
7. » » $. \supset . xa = ax .$
8. $0 = (0, 0, \ldots 0) .$
9. $\mathrm{mod}\, x = \mathrm{m}\, x = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2} .$

$x, y, z \varepsilon \mathrm{q}_n . a, b \varepsilon \mathrm{q} . \supset :$
10. $x + y \varepsilon \mathrm{q}_n .$
11. $x + y = y + x .$
12. $(x + y) + z = x + (y + z) = x + y + z .$
13. $x - x = 0 .$
14. $x + 0 = x .$
15. $ax \varepsilon \mathrm{q}_n .$

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§ 4. 1-31. Grassmann, *Ausdehnungslehre.*
Cayley, *On a theorem relating to the multiple Thetafunctions.* Math. Ann. XVII, pag. 115.

16. $a(x+y) = ax + ay$.
17. $(a+b)x = ax + bx$.
18. $a(bx) = (ab)x = abx$.
19. $1x = x$.
20. $\mathrm{m}\, x \,\varepsilon\, \mathrm{Q}_0$.
21. $\mathrm{mod}(x+y) \leq \mathrm{mod}\, x + \mathrm{mod}\, y$.
22. $\mathrm{mod}\, ax = (\mathrm{mod}\, a)(\mathrm{mod}\, x)$.
23. $\mathrm{m}\, 0 = 0$.
24. $x \mid y = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n$. Def.
25. $x \mid y \,\varepsilon\, \mathrm{q}$.
26. $x \mid x = (\mathrm{m}\, x)^2$.
27. $x \mid y = y \mid x$.
28. $x \mid (y+z) = x \mid y + x \mid z$.
29. $(ax) \mid y = x \mid (ay) = a(x \mid y)$.
30. $\mathrm{i}_1 = (1, 0, 0, \ldots 0) \,.\, \mathrm{i}_2 = (0, 1, 0, \ldots 0) \ldots \mathrm{i}_n = (0, 0 \ldots 0, 1)$. Def.
31. $x = x_1 \mathrm{i}_1 + x_2 \mathrm{i}_2 + \ldots + x_n \mathrm{i}_n$.

$a, b \,\varepsilon\, \mathrm{q} \,.\, a < b \,.\, \supset :$
41. $a \mathbin{\overline{\;\;}} b = (a + \mathrm{Q}) \cap (b - \mathrm{Q})$.
42. $a \mathbin{\vdash\!\dashv} b = (a + \mathrm{Q}_0) \cap (b - \mathrm{Q}_0)$.
43. $a \mathbin{\vdash} b = (a + \mathrm{Q}_0) \cap (b - \mathrm{Q})$. Def.
44. $a \mathbin{\dashv} b = (a + \mathrm{Q}) \cap (b - \mathrm{Q}_0)$.
45. $b \mathbin{\overline{\;\;}} a = a \mathbin{\overline{\;\;}} b \,.\, b \mathbin{\vdash\!\dashv} a = a \mathbin{\vdash\!\dashv} b \,.\, b \mathbin{\dashv} a = a \mathbin{\vdash} b \,.\, b \mathbin{\vdash} a = a \mathbin{\dashv} b$.
46. $\theta = 0 \mathbin{\vdash\!\dashv} 1$.

§ 5. — *D*.

$n \,\varepsilon\, \mathrm{N} \,.\, u, v \,\varepsilon\, \mathrm{Kq}_n \,.\, \supset :$
1. $Du = \mathrm{q}_n \cap \overline{x \varepsilon} \{ \mathrm{l}_1 \mathrm{m} [(u - \iota x) - x] = 0 \}$ Def.
2. $Du = \mathrm{q}_n \cap \overline{x \varepsilon} [h \,\varepsilon\, \mathrm{Q} \,.\, \supset_h .\, \mathrm{num}(u \cap (x + \theta \overline{\mathrm{m}} h)) = \infty]$.
3. $D\mathrm{N} = \Lambda \,.\, D\mathrm{r} = \mathrm{q} \,.\, D\mathrm{q} = \mathrm{q}$.
4. $\mathrm{num}\, u = \infty \,.\, \mathrm{l}' \mathrm{mod}\, u \,\varepsilon\, \mathrm{Q} \,.\, \supset .\, Du - = \Lambda$.
5. $\mathrm{num}\, u \,\varepsilon\, \mathrm{N} \,.\, \supset .\, Du = \Lambda$.

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§ 5. 1, 2, 3. G. Cantor, Math. Ann., V, p. 123 (1871). Acta math., II, p. 343.

4-7. Dini, ib., N. 12, 13.

Cantor, Math. Ann., XV, pag. 1 (1879).

6. $DDu \supset Du$.
7. $p \varepsilon \mathrm{N} . \supset . D^p u \supset Du$.
8. $D(u \cup v) = Du \cup Dv$.
9. $u \supset v . \supset . Du \supset Dv$.
10. $Du \supset u . Dv \supset v . \supset . D(u \cap v) \supset u \cap v$.
11. $Du \supset u . Dv \supset v . \supset . D(u \cup v) \supset u \cup v$.
12. $u \supset Du . v \supset Dv : \supset . u \cup v \supset D(u \cup v)$.
13. $u \supset Du . \supset . Du = D^2 u$.
14. $u \varepsilon \mathrm{Kq} . \mathrm{l}' u \varepsilon \mathrm{q} - u . \supset . \mathrm{l}' u = \max Du$.
14'. » $. \mathrm{l}_1 u$ » $\mathrm{l}_1 u = \min Du$.
15. $a \varepsilon \mathrm{q}_n . \supset . D(a + u) = a + Du$.
16. $(u + Dv) \cup (v + Du) \cup (Du + Dv) \supset D(u + v)$.
17. $D\left(\frac{1}{\mathrm{N}} + \frac{1}{\mathrm{N}}\right) = \frac{1}{\mathrm{N}} \cup \iota 0 . D\left(\frac{1}{\mathrm{N}} - \frac{1}{\mathrm{N}}\right) = \frac{1}{\mathrm{N}} \cup -\frac{1}{\mathrm{N}} \cup \iota 0$.
18. $Du \mathrel{\text{ɔ̄}} u . \supset . \mathrm{num}\, \mathrm{Kq}_n \cap \overline{w \varepsilon} (u = Dw) = \infty$.

21. $D^\omega u = \cap ' D^{\mathrm{N}} u$. Def
22. $D^\omega u = \mathrm{q}_n \cap \overline{x \varepsilon} (p \varepsilon \mathrm{N} . \supset_p . x \varepsilon D^p u)$.
23. $p \varepsilon \mathrm{N} . \supset . D^{p+\omega} u = D^\omega D^p u$. Def.
24. $p \varepsilon \mathrm{N} . \supset . D^{p+\omega} u = D^\omega u$.
25. $p \varepsilon \mathrm{N} . \supset . D^{\omega+p} u = D^p D^\omega u$. Def.
26. $p \varepsilon \mathrm{N} . \supset . D^{p\omega} u = (D^\omega)^p u$. Def.
27. $D^{\omega^2} u = (D^\omega)^\omega u = \cap ' (D^\omega)^{\mathrm{N}} u$. Def.
28. $p \varepsilon \mathrm{N} + 1 . \supset . D^{\omega^p} u = \cap ' (D^{\omega^{p-1}})^{\mathrm{N}} u$. Def.
29. $a, p \varepsilon \mathrm{N} . \supset . D^{a\omega^p} u = (D^{\omega^p})^a u$. Def.
30. $p, a_0, a_1, \ldots a_p \varepsilon \mathrm{N} . \supset . D^{a_0\omega^p + a_1\omega^{p-1} + \ldots + a_{p-1}\omega + a_p} u =$
$D^{a_p} D^{a_{p-1}\omega} \ldots D^{a_1\omega^{p-1}} D^{a_0\omega^p} u$. Def.

8, 18. G. Cantor. Math. Ann., XXIII, pag. 470 (1884).
10, 11, 12. R. De Paolis. *Teoria dei gruppi geometrici,* ecc. Memorie della Società Italiana delle Scienze, 1890, pag. 27, 28.
13. J. Bendixon, Acta mathematica, t. II, 1883, pag. 416.
14, 14'. Dini, ib., N. 16.
21-30. Cantor, Math. Ann., XVII (1880).

$u \varepsilon \mathrm{Kq} . \supset :$

41. $D'u = \mathrm{q} \cap \overline{x \varepsilon} \left[x = \mathrm{l}'(u \cap (x - \mathrm{Q}))\right]$. Def.
42. $D_1 u = \mathrm{q} \cap \overline{x \varepsilon} \left[x = \mathrm{l}_1(u \cap (x + \mathrm{Q}))\right]$. Def.
43. $Du = D'u \cup D_1 u$.
44. $D'(-u) = -D_1 u \,.\, D_1(-u) = -D'u$.
45. $D'(u \cup v) = D'u \cup D'v \,.\, D_1(u \cup v) = D_1 u \cup D_1 v$.
46. $DD'u \supset Du \,.\, DD_1 u \supset Du \,.\, D'Du \supset D'u \,.\, D_1 Du \supset D_1 u \,.\, D'D'u \supset D'u$. $D'D_1 u \supset D'u \,.\, D_1 D'u \supset D_1 u \,.\, D_1 D_1 u \supset D_1 u$.

§ 6. — *I*, *E*, *L*.

$n \varepsilon \mathrm{N} \,.\, u, v \varepsilon \mathrm{Kq}_n . \supset :$

1. $Iu = \mathrm{q}_n \cap \overline{x \varepsilon} (h \varepsilon \mathrm{Q} \,.\, x + \theta\, \overline{\mathrm{m}}\, h \supset u \,.\, -=_h \Lambda)$. Def.
2. $Eu = I(-u)$. Def.
3. $Lu = (-Iu)(-Eu)$. Def.
4. $E(-u) = Iu \,.\, L(-u) = Lu$.
5. $Iu \cap Eu = \Lambda \,.\, Iu \cap Lu = \Lambda \,.\, Eu \cap Lu = \Lambda \,.\, Iu \cup Eu \cup Lu = \mathrm{q}_n$.
6. $Iu \supset u \,.\, Eu \supset -u \,.\, u \supset Iu \cup Lu \,.\, -u \supset Eu \cup Lu$.
7. $IIu = Iu \,.\, IEu = Eu \,.\, Lu = ILu \cup LLu \,.\, LLu = LIu \cup LEu$.
8. $I(u \cap Lu) = \Lambda . ELu = Iu \cup Eu . EIu = -(Iu \cup LIu) . EEu = -(Eu \cup LEu)$.
9. $ILIu = \Lambda \,.\, ILEu = \Lambda \,.\, ILLu = \Lambda \,.\, LLLu = LLu \,.\, LLIu = LIu$. $LLEu = LEu \,.\, LILu \supset LLu$.

11. $u \supset v . \supset . Iu \supset Iv \,.\, Ev \supset Eu \,.\, Lu \supset Iv \cup Lv$.
12. $I(u \cap v) = Iu \cap Iv \,.\, E(u \cup v) = Eu \cap Ev$.
13. $Iu \cup Iv \supset I(u \cup v) \supset Iu \cup Iv \cup (Lu)(Lv)$.
14. $Eu \cup Ev \supset E(u \cap v) \supset Eu \cup Ev \cup (Lu)(Lv)$.
15. $(Iu)(Lv) \cup (Iv)(Lu) \supset L(u \cap v) \supset (Iu)(Lv) \cup (Iv)(Lu) \cup (Lu)(Lv)$.
16. $(Eu)(Lv) \cup (Ev)(Lu) \supset L(u \cup v) \supset (Eu)(Lv) \cup (Ev)(Lu) \cup (Lu)(Lv)$.
17. $I(Iu \cup Iv) = Iu \cup Iv$.
18. $I(LLu \cup LLv) = \Lambda$.
19. $u -= \Lambda \,.\, -u -= \Lambda . \supset . Lu -= \Lambda$.
20. $Iu = u - D(-u)$.

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§ 5. 44-46. Burali-Forti. *Sulle classi derivate a destra e a sinistra.* Atti Acc. Torino, 1894.

§ 6. 1-18. Peano, *Arithmetices principia*, 1889, § 12.

19-20. Jordan, *Cours d'Analyse*, 1893, vol. I, pag. 20.

§ 7. — C, med.

$n \varepsilon \mathrm{N} \,.\, u, v \varepsilon \mathrm{K\,q}_n \,.\, \supset :$

1. $Cu = \mathrm{q}_n \cap \overline{x\,\varepsilon}\,[\mathrm{l}_1\, \mathrm{m}\,(u - x) = 0]\,.$ Def.
2. $Cu = u \cup Du = u \cup Lu = Iu \cup Lu = -\, Eu\,.$
3. $CCu = Cu\,.$
4. $C(u \cup v) = Cu \cup Cv\,.$
5. $u \supset v \,.\supset .\, Cu \supset Cv\,.$
6. $C(u \cap v) \supset Cu \cap Cv\,.$
7. $Cu = u \,.\, Cv = v \,.\supset .\, C(u \cap v) = Cu \cap Cv\,.$
8. $u\, \varepsilon\, \mathrm{Kq} \,.\, \mathrm{l}'u, \mathrm{l}_1 u\, \varepsilon\, \mathrm{q} \,.\supset .\, \mathrm{l}'u, \mathrm{l}_1 u\, \varepsilon\, Cu\,.$
9. $x\, \varepsilon\, Du \,.=.\, x\, \varepsilon\, C(u - \iota x)\,.$
10. $\mathrm{num}\, u\, \varepsilon\, \mathrm{N} \,.\supset .\, u = Cu\,.$

21. $u\, \varepsilon\, \mathrm{K\,q} \,.\supset .\, \mathrm{med}\, u = (\mathrm{l}_1\, u) \frown (\mathrm{l}'\, u)\,.$ Def.
22. » $.\supset .\, \mathrm{med}\, u = \mathrm{q} \cap \overline{x\,\varepsilon}\,(y, z\, \varepsilon\, u \,.\, y < x < z \,.-=_{y,\,z} \Lambda)\,.$

$n \varepsilon \mathrm{N} \,.\, u, v \varepsilon \mathrm{K\,q}_n \,.\, \supset :$

23. $\mathrm{med}\, u = \mathrm{q}_n \cap \overline{x\,\varepsilon}\,(a\, \varepsilon\, \mathrm{q}_n \,.\supset_a .\, a \mid x\, \varepsilon\, \mathrm{med}\,(a \mid u))\,.$ Def.
24. $x, y\, \varepsilon\, u \,.\, x -= y \,.\, p, q\, \varepsilon\, \mathrm{Q} \,.\supset .\, (p\,x + q\,y)/(p + q)\, \varepsilon\, \mathrm{med}\, u\,.$
25. $u \supset v \,.\supset .\, \mathrm{med}\, u \supset \mathrm{med}\, v\,.$
26. $\mathrm{med}\, u = u \,.\, \mathrm{med}\, v = v \,.\supset .\, \mathrm{med}\,(u \cap v) = (\mathrm{med}\, u) \cap (\mathrm{med}\, v)\,.$
27. $\mathrm{med}\, \mathrm{med}\, u = \mathrm{med}\, u\,.$
28. $I\, \mathrm{med}\, u = \mathrm{med}\, u\,.$

G. PEANO.

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§ 7, 1-9. PEANO, *Math. Ann.* XXXVII, pag. 195.