

## VI.

## § 1.

$u, v, w \in K. \circ :$

1.  $u \in v. := f \in (vfu) \text{ sim. } - = f \Delta.$  Def.
2.  $u \in u.$
3.  $u \in v. = . v \in u.$
4.  $u \in v. v \in w. \circ . u \in w.$
5.  $\text{num } u \in N. \circ : u \in v. = . \text{num } u = \text{num } v.$
6.  $\text{num } u = \infty. \circ . v \circ u. v - = u. v \in u : - = v \Delta.$
7.  $n \in N. R \in N. r \in N.$
8.  $\text{Nalg} = q^{\dagger} \in \bar{x} \in (p \in N. a_0, a_1, \dots, a_p \in n. a_0 - = 0. a_0 x^p + a_1 x^{p-1} + \dots + a_p = 0. - = p, a_0, a_1, \dots, a_p \Delta).$  Def.
9.  $\text{Nalg} \in N.$
10.  $u \in N. v \circ u. \text{num } v = \infty. \circ . u \in v.$
11.  $u \in v \in N. \circ . (u \circ v) \in N.$
12.  $u \in Kq. u \in N. a, b \in q. a < b. \circ . (a^{-b}) \circ . (-u) - = \Delta.$

$n \in N. u, v, \dots \in Kq_n. \circ :$

13.  $Du \circ u. \circ . u \in N. \circ . u \in \theta.$
14.  $Du = u. \circ . u \in \theta.$
15.  $u - Du = \Delta. \circ . u \in N.$
16.  $Du \in N. \circ . u \in N.$

## § 1.

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| 1. CANTOR, III, tr. fr., p. 311. | 11. CANTOR, III, tr. fr. p. 313.  |
| 2. DEDEKIND, LIII, n. 32.        | 12. » II, tr. fr., p. 308.        |
| 4. » LIII, n. 33.                | 13. » XXIII, p. 488;              |
| 6. CANTOR, III, tr. fr., p. 311. | » XXV, p. 388.                    |
| 7. » III, tr. fr., p. 319.       | 14. » XXIII, p. 485;              |
| 8. » II, tr. fr., p. 305.        | » XXV, p. 381;                    |
| 9. » II, tr. fr., p. 306.        | BENDIXSON, XXX.                   |
| 10. » III, tr. fr., p. 313.      | 15. CANTOR, XII, tr. fr., p. 373. |
|                                  | 16. » XII, tr. fr., p. 373.       |

17.  $u \in K_{q_n} : h \in Q . \circ h . u \cap \overline{\text{mod } \theta h} \in \text{num } N \cup \infty N . \therefore \circ . u \in \overline{\text{num } N} \cup \infty N .$   
 18.  $\theta - R \infty \theta .$   
 19.  $m, n \in N . \circ . q_m \infty q_n \infty \theta .$   
 20.  $\text{ } . m - = n . \circ . (q_n f q_n) \text{ sim cont} = \Lambda .$   
 21.  $m \in 1 + N . \circ . \therefore f \in (q_m f q) \text{ cont} . f q = q_n : - = f \Lambda .$   
 22.  $u \in \text{Connex} . = : a, b \in u . h \in Q . \circ a, b, h . \therefore p \in N . x \in u \cap Z_p . x_1 = a . x_p =$   
 $= b : r \in Z_{p-1} . \circ r . \text{mod } (x_{r+1} - x_r) < h : - =_p, x \Lambda .$  Def.  
 23.  $u \in \text{Connex} . \circ . u \circ Du .$   
 24.  $u \in \text{Contin} . = : Du \circ u . u \in \text{Connex} .$  Def.  
 25.  $u, v, w, z, \dots \in \text{Connex} : u \cap v - = \Lambda . u \cap w - = \Lambda . u \cap z - = \Lambda \dots : \circ .$   
 $(u \cup v \cup w \cup z \cup \dots) \in \text{Connex} .$   
 26.  $u, v, w, \dots, k \in \text{Contin} : u \cap v - = \Lambda . u \cap w - = \Lambda \dots u \cap k - = \Lambda . \therefore \circ :$   
 $(u \cup v \cup w \cup \dots \cup k) \in \text{Contin} .$   
 27.  $u \in \text{Contin} . v \circ u . v \in N . \circ . u - v \in \text{Connex} .$   
 28.  $u \in Kq . Du \in N . h \in Q . \circ : p \in N . a_1, a_2, \dots, a_p, b_1, b_2, \dots, b_p \in q . u \circ a_1^{-1} b_1 \cup$   
 $a_2^{-1} b_2 \cup \dots \cup a_p^{-1} b_p . \text{mod } (b_1 - a_1) + \text{mod } (b_2 - a_2) + \dots + \text{mod } (b_p - a_p)$   
 $< h . - =_{p, a_1, \dots, a_p, b_1, \dots, b_p} \Lambda .$   
 29.  $Du = u . (Ku) \text{ Contin} = \Lambda . \circ : u = Du . u \cap w = \Lambda . - =_w \Lambda .$   
 30.  $u, v \in Kq . u = Du : a, b \in q . a^{-1} b \circ Du . =_a b \Lambda : v \in N . \therefore \circ :$   
 $c \in q . (u - c) \cap v = \Lambda . - =_c \Lambda .$

## § 2.

1.  $u, v \in K . \circ : Nc^u u = Nc^v v . = . u \infty v .$  Def.  
 2.  $Nc = \overline{x} \in (u \in K . x = Nc^u u : - =_u \Lambda) .$  Def.

17. CANTOR, XXIII, p. 457. 26. DE PAOLIS, LVIII, p. 29  
 18. » III, tr. fr., p. 316. 27. CANTOR, XI, tr. fr., p. 366.  
 19. » III, tr. fr., p. 315. 28. » XII, tr. fr., p. 376.  
 20. THOMAE, V; LUROTH, VI; CANTOR, VII; NETTO, IX; MILESI, LXV. 29. BENDINSON, XX, p. 427.  
 30. SCHEEFFER, XXIX. p. 291.
21. PEANO, LIX; HILBERT, LXI. § 2.  
 22. CANTOR, XVIII, p. 31.  
 23. DE PAOLIS, LVIII, p. 29. 1. CANTOR, XLIX, p. 56.  
 24. CANTOR, XVIII, p. 31. 2. » » p. 56.  
 25. DE PAOLIS, LVIII, p. 28.

3.  $u, v \in K. u \cap v = \Lambda. \circ. Nc'u + Nc'v = Nc'(u \cup v).$  Def.
4.  $a \in Nc. u \in K (K \cap \overline{Nc'a}). Nc'u = b : x, y \in u. x - = y. \circ_{x, y}. x \cap y = \Lambda. \therefore \circ. ab = Nc'(u' \cup u).$  Def.
5.  $a, b, c \in Nc. \circ. a + b = b + a. a + (b + c) = (a + b) + c. ab = ba. a(bc) = (ab)c. a(b + c) = ab + ac.$
6.  $a, b, c \in Nc. \circ. a + b + c = a + (b + c) = (a + b) + c. abc = a(bc) = (ab)c.$  Def.
7.  $u \in Kord. = \therefore u \in K : x, y \in u. \circ. x = y \cup x S_u y \cup y S_u x : x, y \in u. x S_u y. y S_u x. =_{x, y} \Lambda : x, y, z \in u. x S_u y. y S_u z. \circ_{x, y, z}. x S_u z.$  Def.
8.  $u, v \in Kord. \circ : f \in (v \text{ ford } u). = \therefore f \in (v f u) \text{ sim} : x, y \in u. x S_u y. \circ. f x S_v f y.$  Def.
9.  $u \in K \text{ bord}. = : u \in Kord. \overline{x \in (x \in u : y \in u. y S_u x. =_y \Lambda)} - = \Lambda : x \in u. \circ_{x, y}. y \in u. x S_u y. \overline{z \in (z \in u. x S_u z. z S_u y)} = \Lambda : - =_y \Lambda.$  Def.
10.  $u, v \in K \text{ bord}. \circ : Ntrasf'u = Ntrasf'v. = f \in (v \text{ ford } u). - =_f \Lambda.$  Def.
11.  $Ntrasf = \overline{x \in (u \in K \text{ bord}. x = Ntrasf'u. - =_u \Lambda)}.$  Def.
12.  $u, v \in K \text{ bord}. Ntrasf'u = Ntrasf'v. \circ. u \subset v.$
13.  $u, v \in K \text{ bord}. \circ : Ntrasf'u > Ntrasf'v. = \therefore Ntrasf'u - = Ntrasf'v. w \in K \text{ bord}. w \circ u. Ntrasf'w = Ntrasf'v. - =_w \Lambda.$  Def.
14.  $\alpha, \beta \in Ntrasf. \circ : \beta < \alpha. =. \alpha > \beta.$  Def.
15.  $\alpha, \beta \in Ntrasf. \circ : \alpha = \beta. \cup. \alpha > \beta. \cup. \alpha < \beta.$
16.  $m \in N. \circ : u = Y_m. = \therefore u \in K \text{ bord } N_0 : 0 \in u : m' \in N. m' \rightarrow m. =_{m'}. m' \in u : m', m'' \in u. m' < m''. \circ_{m', m''}. m' S_u m''.$  Def.
17.  $m \in N. \circ. Ntrasf' Y_m = m + 1 = \text{num } Y_m.$  Def.
18.  $u \in K \text{ bord}. \text{num } u \in N. \circ. Ntrasf' u = \text{num } u.$
19.  $u \in K \text{ bord}. \text{num } u = \infty. m \in N. \circ. Ntrasf' u > m.$
20.  $u = \omega. = \therefore u \in Ntrasf : m \in N. \circ_{m, u}. u > m : \alpha \in (Ntrasf - N). \alpha < u. = \Lambda.$  Def.
21.  $\alpha \in Ntrasf. \circ : u = Y_\alpha. = \therefore u \in K \text{ bord } Ntrasf : 0 \in u : \alpha' \in Ntrasf. \alpha' \rightarrow \alpha. =_{\alpha'}. \alpha' \in u : \alpha', \alpha'' \in u. \alpha' < \alpha''. \circ_{\alpha', \alpha''}. \alpha' S_u \alpha''.$  Def.
22.  $\alpha \in Ntrasf. \circ. Ntrasf'(Y_\alpha - \alpha) = \alpha.$  Def.

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| 3. CANTOR, XLIX, p. 59.   | 13. CANTOR, XLIX, p. 26. |
| 4. " " p. 60.             | 15. " " p. 26.           |
| 5. " " p. 59, 60.         | 17. " XVIII, p. 3.       |
| 7. GUTBERLET, XL, p. 183. | 18. " " p. 3.            |
| 9. CANTOR, XVIII, p. 4.   | 20. " " p. 33;           |
| 10. " " p. 5.             | " XLIX, p. 34.           |
| 12. " XLIX, p. 74.        | 22. " XVIII, p. 15.      |

23.  $u, v \in K \text{ bord} . u \cap v = \Lambda . \therefore w \in K \text{ ord} : w = u \cup v : x, y \in u . x S_u y .$   
 $\circ_{x, y} . x S_v y : x, y \in v . x S_v y . \circ_{x, y} . x S_w y : x \in u, y \in v . \circ_{x, y} . x S_w y ::$   
 $\circ . w \in K \text{ bord} .$
24. » » » Ntrasf'  $u = \alpha$ . Ntrasf'  $v = \beta$ :  $\circ$ . Ntrasf'  $w = \alpha + \beta$ .  
 Def.
25.  $v \in K \text{ bord} : u \in v . \circ_u . u \in K \text{ bord} : u, u' \in u . \circ_{u, u'} . u \cap u' = \Lambda :: w \in K \text{ ord}$   
 $\therefore x \in u . u \in v . = x \in w : u \in v . x, y \in u . x S_u y . \circ_{x, y, u} . x S_w y :$   
 $u, u' \in v . u S_v u' . x \in u . y \in u' . \circ_{x, y, u, u'} . x S_w y :: \circ : w \in K \text{ bord} .$
26. » » » Ntrasf'  $v = \beta : u \in v . \circ_u .$  Ntrasf'  $u = \alpha . \therefore \circ$ .  
 Ntrasf'  $w = \alpha \beta$ .  
 Def.
27.  $\alpha \in \text{II} . = : \alpha \in \text{Ntrasf} . Y_\alpha \simeq N$ .  
 Def.
28.  $\text{II} - \infty N$ .
29.  $u \in K \text{ II} . u \in N . \max u = \Lambda : \circ . \beta \in (\beta \in \text{II} . u \circ Y_\beta : \beta' \in (Y_\beta - \beta) . \circ \beta' .$   
 $u - \circ Y_{\beta'}) . - = \Lambda$ .
30.  $\beta \in K (N \cup \text{II}) . \alpha \beta \in K \text{ II} : \beta' , \beta'' \in \beta . \beta' < \beta'' . \circ \beta' , \beta'' . \alpha \beta' > \alpha \beta'' . \therefore \circ$   
 $\text{num } \alpha \beta \in N$ .
31.  $u \in K \text{ II} . \circ . \min u = \Lambda$ .
32.  $u \in K \text{ II} . \circ : \text{num } u \in N . \cup . u \simeq N . \cup . u \simeq \text{II}$ .
33.  $u \in K . u \in N . \alpha \in \text{II} : \circ_\alpha : v \in K \text{ bord} . v = u . \text{Ntf}' v = \alpha . - = v \Lambda$ .
34.  $\alpha, \beta, \gamma \in N \cup \text{II} . \circ : \alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma . \alpha (\beta \gamma) = (\alpha \beta) \gamma .$   
 $\alpha (\beta + \gamma) = \alpha \beta + \alpha \gamma$ .
35.  $\alpha, \beta \in N \cup \text{II} . \alpha + \beta - = \beta + \alpha . - = \alpha, \beta \Lambda$ .  
 $\circ . \alpha \beta - = \beta \alpha . - = \alpha, \beta \Lambda$ .
36.  $\alpha, \beta, \gamma \in N \cup \text{II} . \circ : \alpha + \beta + \gamma = \alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma . \alpha \beta \gamma =$   
 $\alpha (\beta \gamma) = (\alpha \beta) \gamma$ .  
 Def.
37.  $u = \Omega . = . \therefore u \in \text{Ntrasf} : \text{II} \circ Y_u : \beta \in \text{Ntrasf} . \beta < u . \text{II} \circ Y_\beta . = \Lambda$ .  
 Def.
38.  $u \in K \text{ ord}_n q_n . = : u \in K q_n . \text{El}_1 u \in K \text{ ord} . \text{El}_2 u \in K \text{ ord} \dots \text{El}_n u \in K \text{ ord} .$   
 Def.

23. CANTOR, XVIII, p. 6.

24. » » »

25. » » » p. 7.

26. » » »

27. » » » p. 35.

28. » » »

29. » » »

30. » » » p. 37.

31. CANTOR, XVIII, p. 37.

32. » » » p. 38.

33. » » » p. 5.

34. » » » p. 7, 39;

» XLIX, p. 26.

35. » XVIII, p. 6, 7.

37. » » » p. 38.

38. » XLIX, p. 68.

39.  $u, v \in \text{Kord} . \circ . \text{Ty}_1' u = \text{Ty}_1' v . = . f \varepsilon (v \text{ f ord } u) - = f \Lambda .$  Def.
40.  $u, v \in \text{Kord}_n . \circ : \text{Ty}_n' u = \text{Ty}_n' v . = . \text{Ty}_1' \text{El}_1 u = \text{Ty}_1' \text{El}_1 v . \text{Ty}_1' \text{El}_2 u = \text{Ty}_1' \text{El}_2 v \dots \text{Ty}_1' \text{El}_n u = \text{Ty}_1' \text{El}_n v .$  Def.
41.  $\text{Ty}_n = \overline{u \varepsilon \text{Kord}_n . x = \text{Ty}_n' u . - = u \Lambda} .$  Def.
42.  $u, v \in \text{Kord}_n \text{q}_n . u \cap v = \Lambda . w \in \text{Kord}_n \text{q}_n . w = u \cup v : x, y \varepsilon u . i \varepsilon Z_n . x_i S_u y_i . \circ_{x,y,i} . x_i S_w y_i : x, y \varepsilon v . i \varepsilon Z_n . x_i S_v y_i . \circ_{x,y,i} . x_i S_w y_i : x \varepsilon u . y \varepsilon v . i \varepsilon Z_n . \circ_{x,y,i} . x_i S_w y_i : \text{Ty}_n' u = \alpha . \text{Ty}_n' v = \beta . \text{Ty}_n' w = \gamma . \circ . \gamma = \alpha + \beta .$  Def.
43.  $v \in \text{Kord}_n . \text{Ty}_n' v = \beta : u \varepsilon v . \circ_u . u \in \text{Kord}_n \text{q}_n . \text{Ty}_n' u = \alpha : u, u' \varepsilon v . \circ_u . u' u \cap u' = \Lambda : w \in \text{Kord}_n \text{q}_n . \text{Ty}_n' w = \gamma : x \varepsilon u . u \varepsilon v . = . x \varepsilon w : u' \varepsilon v . x, y \varepsilon u' . i \varepsilon Z_n . x_i S_{u'} y_i . \circ_{u',x,y,i} . x_i S_w y_i : u', u'' \varepsilon v . x \varepsilon u' . y \varepsilon u'' . i \varepsilon Z_n . u'_i S_v u''_i . \circ_{u',u'',x,y,i} . x_i S_w y_i : \circ . \gamma = \alpha \beta .$  Def.
44.  $\alpha, \beta, \gamma \varepsilon \text{Ty}_n . \circ : \alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma . \alpha (\beta \gamma) = (\alpha \beta) \gamma . \alpha (\beta + \gamma) = \alpha \beta + \alpha \gamma .$
45.  $\alpha, \beta \varepsilon \text{Ty}_n . \alpha + \beta - = \beta + \alpha . - = \alpha, \beta \Lambda .$   
 $\alpha \beta - = \beta \alpha . - = \alpha, \beta \Lambda .$
46.  $\alpha, \beta, \gamma \varepsilon \text{Ty}_n . \circ : \alpha + \beta + \gamma = \alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma . \alpha \beta \gamma = \alpha (\beta \gamma) = (\alpha \beta) \gamma .$  Def.
47.  $m \varepsilon N . \circ . \Phi (m, n) = \text{num } \overline{\alpha \varepsilon \text{Ty}_n : u \varepsilon \text{Kord}_n . \text{num } u = m . \circ_u . \text{Ty}_n' u = \alpha} .$  Def.
48.  $\Phi (m, n) = \sum_{\substack{\gamma_i=0,1,\dots,1 \\ i=1,2,\dots,n \\ i=1,2,\dots,m}} (-1)^{i_1+i_2+\dots+i_n} \binom{1_1}{\gamma_1} \binom{1_2}{\gamma_2} \dots \binom{1_n}{\gamma_n} \binom{(1_1-\gamma_1)(1_2-\gamma_2)\dots(1_n-\gamma_n)}{m} .$

§ 3.

1.  $\gamma \varepsilon I \cup II . D^{\gamma} u \in N . \circ . u \in N .$
2.  $u \in N . Du \circ u . \circ . : u = v \cup w . v \in N . w = Dw . \gamma \varepsilon I \cup II . D^{\gamma} u = w : - = v, w, \gamma \Lambda .$

39. CANTOR, XLIX, p. 71.

40. » » »

41. » » »

42. » » p. 76.

43. » » p. 77.

44. » » p. 76, 78.

45. » » »

47. CANTOR, XLIX, p. 82.

48. SCHWARZ, LI, p. 11.

§ 3.

1. CANTOR, XII, tr. fr., p. 376.

2. » XXIII, p. 471.

$$3. Du \in N. \circ : \gamma \in I \cup II. D^\gamma u = \Lambda. - =_\gamma \Lambda.$$

$$4. \gamma \in I \cup II. D^\gamma u = \Lambda. \circ. u \in Du \in N.$$

$$5. Du \in N. \circ. \overline{x \in (\gamma \in I \cup II. \circ_\gamma. x \in D^\gamma u)} - = \Lambda.$$

$$6. D^\Omega u = \overline{x \in (\gamma \in I \cup II. \circ_\gamma. x \in D^\gamma u)} = \cap \{ D^N \cup II u \}. \quad \text{Def.}$$

$$7. Du - D^\Omega u \in N.$$

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3. CANTOR, XVIII, p. 7-8, 31.

6. BENDIXSON, XX, p. 419.

4. » » »

7. » » »

5. BENDIXSON, XX, p. 419.

PHRAGMÉN, XXVI.