

279.

ON A THEOREM RELATING TO SPHERICAL CONICS.

[From the *Quarterly Journal of Pure and Applied Mathematics*, vol. III. (1860), p. 53.]

THE following theorem was given by Prof. Maccullagh: "If three lines at right angles to each other pass through a fixed point O so that two of them are confined to given planes: the third line traces out a cone of the second order whose sections parallel to the given planes are circles, and the plane containing the other two lines envelopes a cone of the second order whose sections by planes parallel to the given planes are parabolas."

Referring the figure to the sphere we have a trirectangular triangle XYZ , of which two angles X, Y lie on fixed arcs A, B . The angle Z generates a spherical conic U' having A, B for its cyclic arcs. The side XY envelopes a spherical conic U touched by the arcs A, B . The conic U' is evidently the supplementary conic of U , hence the poles of A, B are the foci of U . We may drop altogether the consideration of the triangle XYZ and consider only the side XY , we have then the theorem:

If a quadrantal arc XY slides between the two fixed arcs A, B , the envelope of XY is a spherical conic U touched by the fixed arcs A, B , and which has for its foci the poles of these same arcs A, B .

It is worth while to notice the great reduction of order which takes place in consequence of the arc XY being a quadrant. If XY had been an arc of a given magnitude θ , the envelope would have been a spherical curve of an order certainly higher than 6. For considering the corresponding problem in plano, the envelope in the particular case where the fixed lines A, B are at right angles to each other is a curve of the sixth order, and in the general case where the two fixed lines are not at right angles the order is higher: the problem in plano corresponds of course, not to the general problem on the sphere, but to that in which θ is indefinitely small.