

XVII.

THE AUXILIARY FUNCTION T FOR A TELESCOPE, WHEN THE AXIS OF EYEPIECE IS NOT COINCIDENT WITH, BUT PARALLEL TO, THAT OF OBJECT GLASS

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[Note Book 28, pp. 17, 19 (back).]

 σ, τ , small emergent direction cosines; σ', τ' , incident; s, t , intermediate;

$$T_o = T_o^{(0)} + P_o(s^2 + t^2) + P_{o'}(s\sigma' + t\tau') + P_o'(\sigma'^2 + \tau'^2) + T_o^{(4)},$$

$$T_e = T_e^{(0)} + P_e(\sigma^2 + \tau^2) + P_{e'}(s\sigma + t\tau) + P_e'(\sigma^2 + \tau^2) + T_e^{(4)} + a(\sigma - s) + b(\tau - t),$$

$$T = T_o + T_e;$$

$T_o^{(0)}, P_o, P_{o'}, P_o'$, are constants for object glass, and $T_e^{(0)}, P_e, P_{e'}, P_e'$, are corresponding constants for eye glass; a, b , are small coordinates of axis of eye glass, referred to an origin of coordinates on the axis of the object glass;* the terms $T_o^{(4)}, T_e^{(4)}$, in T_o, T_e , are of the 4th and higher dimensions, and we may put with sufficient approximation (neglecting 6th dimension)

$$T_o^{(4)} = Q_o(s^2 + t^2)^2 + Q_{o'}(s^2 + t^2)(s\sigma' + t\tau') + Q_o''(s^2 + t^2)(\sigma'^2 + \tau'^2) + Q_{o'''}(s\sigma' + t\tau')^2 + Q_{o''''}(s\sigma' + t\tau')(\sigma'^2 + \tau'^2),$$

$$T_e^{(4)} = Q_e(\sigma^2 + \tau^2)^2 + Q_{e'}(\sigma^2 + \tau^2)(s\sigma + t\tau) + Q_e''(\sigma^2 + \tau^2)(s^2 + t^2) + Q_{e'''}(s\sigma + t\tau)^2 + Q_{e''''}(s\sigma + t\tau)(s^2 + t^2).$$

To determine s, t , as functions of $\sigma, \tau, \sigma', \tau'$, and of the constants, we have to make $T_o + T_e$ a minimum or a maximum;† that is, we are to put

$$0 = 2(P_o + P_o')s + P_{e'}\sigma + P_{o'}\sigma' - a,$$

$$0 = 2(P_e + P_e')t + P_{e'}\tau + P_{o'}\tau' - b,$$

if we continue to neglect terms of the 6th dimension in T .

* [For any given optical system, the auxiliary function T is a function of the initial σ', τ', ν' , and of the final σ, τ, ν , but the form of this function will depend not only on the orientation of the axes of coordinates, but also on the position of the origin. The general transformation of T for change of origin is easily obtained. Let x', y', z', x, y, z , be the coordinates of initial and final points referred to certain axes $Oxyz$, and let $\bar{x}', \bar{y}', \bar{z}', \bar{x}, \bar{y}, \bar{z}$, be the coordinates of the same points referred to parallel axes $\bar{O}\bar{x}\bar{y}\bar{z}$, where \bar{O} is $x=a, y=b, z=c$. Then we have $x - \bar{x} = x' - \bar{x}' = a, y - \bar{y} = y' - \bar{y}' = b, z - \bar{z} = z' - \bar{z}' = c$. This transformation does not change the values of $\sigma', \tau', \nu', \sigma, \tau, \nu$, for $\bar{\sigma} = \partial \bar{V} / \partial \bar{x} = \partial V / \partial x = \sigma$. Let T, \bar{T} , denote the values of the auxiliary function corresponding to the two sets of axes. Then

$$T = \sigma x + \tau y + \nu z - \sigma' x' - \tau' y' - \nu' z' - V = \bar{\sigma} \bar{x} + \bar{\tau} \bar{y} + \bar{\nu} \bar{z} - \bar{\sigma}' \bar{x}' - \bar{\tau}' \bar{y}' - \bar{\nu}' \bar{z}' - \bar{V} + a(\sigma - \sigma') + b(\tau - \tau') + c(\nu - \nu'),$$

since V is invariant ($V = \bar{V}$). Thus

$$T = \bar{T} + a(\sigma - \sigma') + b(\tau - \tau') + c(\nu - \nu').$$

When we make use of the symmetry of the eye-piece about its axis, we see at once that this formula of transformation, with $\sigma' = s, \tau' = t, c = 0$, yields the given expression for T_e . The formula is exact, i.e. no terms are neglected on account of the smallness of a and b .]

† [Cf. pp. 217, 218.]

For the approximate image X, Y, Z , of a star σ', τ' , formed by this uncentred telescope, we have

$$X = Z\sigma + a + 2P_e\sigma + P_e s = a + \frac{P_{ie}(a - P_{io}\sigma')}{2(P_o + P'_e)} + \sigma \left\{ Z + 2P_e - \frac{P_{ie}^2}{2(P_o + P'_e)} \right\},$$

$$Y = b + \frac{P_{ie}(b - P_{io}\tau')}{2(P_o + P'_e)} + \tau \left\{ Z + 2P_e - \frac{P_{ie}^2}{2(P_o + P'_e)} \right\},$$

independently of σ, τ ; that is,

$$\left. \begin{aligned} X &= a + \frac{P_{ie}(a - P_{io}\sigma')}{2(P_o + P'_e)}, \\ Y &= b + \frac{P_{ie}(b - P_{io}\tau')}{2(P_o + P'_e)}, \\ Z &= -2P_e + \frac{P_{ie}^2}{2(P_o + P'_e)}. \end{aligned} \right\} \begin{array}{l} \text{Image of a star, formed by telescope} \\ \text{with two parallel axes, an eye axis} \\ \text{and an object axis.} \end{array}$$

The image will therefore be on axis of eyepiece if $a = P_{io}\sigma', b = P_{io}\tau'$, that is, if this axis pass through the image of the star formed by the object glass.

Let $a = P_{io}\sigma' + A\sigma'(\sigma'^2 + \tau'^2), \quad b = P_{io}\tau' + A\tau'(\sigma'^2 + \tau'^2),$

so that the star shall be in the plane of the two parallel axes, and so that its image formed by the telescope shall be nearly on the axis of the eyepiece. Then in $T = T_o + T_e$, if we put

$$\lambda = -\frac{P_{ie}}{2(P_o + P'_e)},$$

we shall have

$$T = T_o^{(0)} + T_e^{(0)} + P_e(\sigma^2 + \tau^2) + P'_o(\sigma'^2 + \tau'^2) + a\sigma + b\tau + T_o^{(4)} + T_e^{(4)}$$

$$- A(\sigma\sigma' + \tau\tau')(\sigma'^2 + \tau'^2) + (P_o + P'_e)\{(s - \lambda\sigma)^2 + (t - \lambda\tau)^2 - \lambda^2(\sigma^2 + \tau^2)\},$$

$$s = \lambda\sigma + \text{terms of 3rd dimension,}$$

$$t = \lambda\tau + \text{terms of 3rd dimension;}$$

therefore, neglecting 6th dimension,

$$T = T_o^{(0)} + T_e^{(0)} + (\sigma^2 + \tau^2)P_e - \frac{P_{ie}^2}{4(P_o + P'_e)} \left\} + P_{io}(\sigma\sigma' + \tau\tau') + P'_o(\sigma'^2 + \tau'^2) + T^{(4)};$$

$$T^{(4)} = T_o^{(4)} + T_e^{(4)} + A(1 - \lambda)(\sigma\sigma' + \tau\tau')(\sigma'^2 + \tau'^2)$$

$$= Q(\sigma^2 + \tau^2)^2 + Q_1(\sigma^2 + \tau^2)(\sigma\sigma' + \tau\tau') + Q'(\sigma^2 + \tau^2)(\sigma'^2 + \tau'^2)$$

$$+ Q_{11}(\sigma\sigma' + \tau\tau')^2 + Q'_1(\sigma\sigma' + \tau\tau')(\sigma'^2 + \tau'^2) + Q''(\sigma'^2 + \tau'^2)^2;$$

$$\left\{ \begin{aligned} Q &= Q_e + \lambda Q_{ie} + \lambda^2(Q'_e + Q_{11e}) + \lambda^3 Q'_{ie} + \lambda^4(Q''_e + Q_o); \\ Q_1 &= \lambda^3 Q_{io}; \quad Q_{11} = \lambda^2 Q_{11o}; \quad Q'' = Q''_o; \\ Q' &= \lambda^2 Q'_o; \quad Q'_1 = \lambda Q'_{io} + A(1 - \lambda). \end{aligned} \right.$$