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On the dynamic flow of granular media

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STATISTICAL methods, developed for the description of the turbulent flow of fluids, are employed for describing granular media flow irregularities connected with the final dimensions of grains. The general form of the differential equations of motion is obtained in terms of mean quantities.

Do opisu nieregularności przepływu ośrodków sypkich spowodowanych skończonymi wymiarami ziaren zastosowano metody statystyczne opisu zjawisk turbulencji w cieczach. Otrzymano ogólną postać różniczkowych równań ruchu, wyrażonych przez wielkości uśrednione.

Статистические методы описания турбулентного течения жидкостей применяются для описания, вызванных конечными размерами зерен, возмущений течения сыпучей среды. Получены, в общем виде, дифференциальные уравнения осредненного движения среды.

1. Introduction

IN THE FRAMEWORK of most theories devoted to the continuous description of flow problems of granular media, these media are considered in fact, as fluids with very special propetries. If only a model of such a fluid specified, the further description of the flow as a rule is confined to the flow patterns with well-defined streamlines. Thus, such an approach is related to the description of laminar flow of "true" fluids. However, a simple observation of the real flow of granular media can lead to quite a different conclusion. We can easily see that any material point e.g. a chosen point of any individual grain moves in a very complex way due to the rotation of the finite dimension grains and also due to the irregular Brownian — like motion of each grain as a whole. For these reasons, motion of the material particle seems to be more similar to the motion to the material point in the case of the turbulent flow of "true" fluids.

The present paper is an attempt to employ some elementary concepts of the turbulent flow theory for the description of flow of granular media.

In the development of the theories of turbulent flow most authors used two- or more-point correlation functions [1]. However, some authors (see e.g. [2, 3, 4, 5]) prefer to deal with one-point correlations only. Such an approach leads to the semi-phenomenological theories of anisotropic unsteady turbulent flow. It should be mentioned, however, that the results obtained in this way are sometimes so complex that the practical application or even the experimental verification of the model are in fact not possible.

It can be expected that for granular media the inertial effects due to the irregular small motions are less significant than for liquids. It seems also reasonable to expect that dissipation can play a more significant role in our case than in that of liquids. These considerations allow us to assume that reasonable accuracy in our case can be achieved by considering only the correlation moments of the lower (not exceeding three) order.

On the other hand, however, the situation seems to be more complex than in usually considered cases of incompressible fluids with constant viscosity, namely:

1) velocity, density and stress fields are in our case not differentiable and not even continuous functions;

2) in contrary to the theory of the turbulent flow of liquids we are not able to assume any reasonable constitutive equation on the level of small scale motion;

3) it seems rathers unreasonable to assume the large scale incompressibility of the medium.

2. Averaging, mean values

We consider the set of moving grains of finite dimensions and assume that all grains are made of the same material with constant density, i.e. we assume incompressibility of grains (but we do not assume that they are rigid). We assume also that the space between the grains is empty. It is our aim to describe the average motion, i.e. we are to find the relations between the average values of the density, velocity and stress fields.

By the term "average" we shall always understand ensemble average, and assume that the average fields are continuous and sufficiently smooth. Of course, in the future some kind of the ergodic theorem is necessary here, but we are not going to discuss this problem in details in the present paper.

Let us consider a chosen realization of the particular flow; any random field can be expressed as the sum of the average term and fluctuation term:

$$\begin{aligned} \varrho(\mathbf{x}, t) &= \overline{\varrho(\mathbf{x}, t)}+\varrho'(\mathbf{x}, t), \\ \mathbf{u}(\mathbf{x}, t) &= \overline{\mathbf{u}(\mathbf{x}, t)}+\mathbf{u}'(\mathbf{x}, t), \\ \mathbf{T}(\mathbf{x}, t) &= \overline{\mathbf{T}(\mathbf{x}, t)}+\mathbf{T}'(\mathbf{x}, t), \end{aligned} \tag{2.1}$$

by definition $\overline{\varrho'(\mathbf{x}, t)} = 0$, $\overline{\mathbf{u}'(\mathbf{x}, t)} = 0$, $\overline{T'(\mathbf{x}, t)} = 0$ where ϱ is the density, $\mathbf{u}$ — velocity and $\mathbf{T}$, $(\mathbf{T}^T = \mathbf{T})$ is the stress.

Now the question arises as to the value of the velocity field for such $\mathbf{x}$ that $\varrho(\mathbf{x}) = 0$. Of course, for zero density we can assume any value of the velocity vector; however, we will show that there is one particular choice which, not affecting the physical sense of the formulae, makes them more compact and clear.

Let us consider the mean value of the momentum:

$$\overline{\varrho u_i} = \overline{P}_i = \overline{\overline{P}_i + P'_i}, \text{ (}^1\text{)} \tag{2.2}$$

on the other hand we can write

$$\overline{\varrho u_i} = \overline{(\overline{\varrho}+\varrho')(\overline{u}_i+u'_i)} = \overline{\varrho}\,\overline{u}_i+\overline{\varrho' u'_i}. \tag{2.3}$$

It is seen that in general $\overline{P}_i \neq \overline{\varrho}\,\overline{u}_i$.

For granular media the quantity expressed as $n_i\overline{\varrho u_i}$ is one of the most important characteristics, being the measure of the mean mass flux across a unit area normal to $\mathbf{n}$, on

(1) Throughout the whole paper we use the Cartesian frame only.

the other hand, the physical significance of the quantity $\bar{\varrho}$ is also obvious. The quantity $\bar{u}_i$ depends on the choice of the velocity field for the points for which $\varrho = 0$.

It can be easily shown in the case of an ensemble of a finite number of realizations that when choosing $u_i = \bar{u}_i$ for these points, we obtain

$$\bar{P}_i = \bar{\varrho}\bar{u}_i. \tag{2.4}$$

Indeed, let $m+n$ be the number of realizations, and for the chosen point $\mathbf{x}$ and chosen time instant t for m realizations, let $\varrho(\mathbf{x}) = \varrho_0(\mathbf{x})$ where $\varrho_0(\mathbf{x})$ denotes the density of grain material, let n denote the number of realizations for which $\varrho(\mathbf{x}) = 0$, then,

$$\bar{\varrho} = \frac{1}{m+n}\sum_{\alpha=1}^{m+n}\varrho_{(\alpha)} = \frac{m}{m+n}\varrho_0, \tag{2.5}$$

$$\bar{u}_i = \frac{1}{m+n}\Big(\sum_{\alpha=1}^{m}u_{i(\alpha)} + \sum_{\beta=1}^{n}u_{i(\beta)}\Big) = \frac{1}{m+n}\Big(\sum_{\alpha=1}^{m}u_{i(\alpha)} + n\bar{u}_i\Big), \tag{2.6}$$

$$\bar{u}_i = \frac{m}{n+m}\langle u_i\rangle + \frac{n}{n+m}\bar{u}_i, \tag{2.7}$$

where by $\langle u_i\rangle$ we denote the mean velocity for these realizations for which $\varrho \neq 0$. We can see that taking $u_i = \bar{u}_i$ for these points $\mathbf{x}$ for which $\varrho(\mathbf{x}) = 0$, we obtain $\bar{u}_i = \langle u_i\rangle$; for the quantity $\overline{\varrho u_i}$ we have

$$\overline{\varrho u_i} = \frac{1}{m+n}\sum_{\alpha=1}^{m}u_{i(\alpha)}\varrho = \frac{m}{m+n}\varrho_0\langle u_i\rangle = \bar{\varrho}\bar{u}_i, \tag{2.8}$$

i.e. in our case

$$\overline{\varrho' u_i'} = 0, \tag{2.9}$$

and our choice yields the simplest form of the mean momentum. By similar considerations we also arrive at the following relations:

$$\overline{\varrho u_i u_k} = \bar{\varrho}\bar{u}_i\bar{u}_k + \bar{\varrho}\langle u_i' u_k'\rangle, \tag{2.10}$$

$$\overline{\varrho u_i u_k u_l} = \bar{\varrho}\bar{u}_i\bar{u}_k\bar{u}_l + \bar{\varrho}\langle u_i' u_k' u_l'\rangle + \bar{\varrho}\bar{u}_i\langle u_k' u_l'\rangle + \bar{\varrho}\bar{u}_k\langle u_i' u_l'\rangle + \bar{\varrho}\bar{u}_l\langle u_i' u_k'\rangle, \tag{2.11}$$

where brackets denote the same operation as in Eq (2.7). For the sake of brevity we denote

$$\bar{\varkappa}_{ij} \overset{\text{df}}{=} \langle u_i' u_j'\rangle,$$
$$\overline{\beta_{ijk}} \overset{\text{df}}{=} \langle u_i' u_j' u_k'\rangle.$$

3. Equations of motion and evolution

We assume now that for fixed region in space containing grains of material the following balance laws are valid:

a) balance of mass

$$\frac{\partial}{\partial t}\int_V \varrho dV = -\int_{\partial V}\varrho u_j n_j dS, \tag{3.1}$$

b) balance of momentum

$$\frac{\partial}{\partial t}\int\limits_V \varrho u_i dV = \int\limits_{\partial V} T_{ij}n_j dS - \int\limits_{\partial V} \varrho u_i u_j n_j dS, \tag{3.2}$$

c) balance of angular momentum

$$\frac{\partial}{\partial t}\int\limits_V \varrho u_j x_k \varepsilon_{ijk} dV = \int\limits_{\partial V} T_{jl}n_l x_k \varepsilon_{ijk} dS - \int\limits_{\partial V} \varrho u_j u_l n_l x_k \varepsilon_{ijk} dS, \text{(}^2\text{)}. \tag{3.3}$$

In Eqs. (3.1) to (3.3) n_j denotes the component of the unit normal vector. ε_{ijk} is the permutation symbol and T_{ij} is the symmetric stress tensor, i.e. we assume that the material of grains is a classical material. The assumptions (3.1) to (3.3) are rather obvious: if the fields under consideration were differentiable, we would easily obtain from Eqs. (3.1) to (3.3) usual differential equations of continuous media.

For continuous media we can also write the following balance law:

$$\frac{\partial}{\partial t}\int\limits_V \varrho u_i u_k dV = 2\int\limits_{\partial V} (T_{ij}u_k)^{(i,k)} n_j dS - \int\limits_{\partial V} \varrho(u_i u_j u_k) n_j dS - 2\int\limits_V (T_{ij}u_{k,j})^{(i,k)} dV, \tag{3.4}$$

where $A^{(ik)}_{ik...l}$ denotes the symmetric part of the tensor: $\frac{1}{2}(A_{ik...l}+A_{ki...l})$. Equation (3.4) is not independent, it can be easily obtained from Eq. (3.2); taking the trace of both sides of Eq. (3.4) and multiplying by 1/2 we can obtain the energy conservation law. For these materials which are incapable of energy storing, e.g. for perfectly plastic materials, the one half of the trace of the last term is equal to the dissipation rate.

For the case of discontinuous velocity and stress fields the last term has no mathematical meaning, even in the class of generalized functions. But it is reasonable to assume that, even in this case, it has a certain physical significance. So we can make the following assumption: let for every spatial region such a balance law hold

$$\frac{\partial}{\partial t}\int\limits_V \varrho u_i u_k dV = \int\limits_{\partial V} (T_{ij}u_k + T_{kj}u_i) n_j dS - \int\limits_{\partial V} \varrho(u_i u_j u_k) n_j dS + \int\limits_V Д_{ik} dV, \tag{3.5}$$

where $Д_{ij}$ is a symmetric tensor for which we have

$$\frac{1}{2}Д_{ii} = D \geqslant 0, \tag{3.6}$$

where D is the dissipation rate per unit volume. (We excluded in this way elastic grains from further considerations confining ourselves to the perfectly plastic or rigid grains.) Looking from another viewpoint we can consider Eq. (3.5) as a definition of the tensor $Д_{ij}$.

It seems reasonable to call $Д_{ij}$ a dispersivity tensor.

The only assumption about the regularity of $Д_{ij}$ is its integrability. We assume of course that $\overline{Д}_{ij}$ is sufficiently smooth.

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(2) We neglected here, for the sake of brevity, the action of mass forces; the introduction of deterministic mass forces does not change our considerations, introducing merely one additional term.

Expressing all the fields in Eqs. (3.1) to (3.3) and Eq. (3.4) by the mean values and fluctuation, and averaging, we obtain integral expressions containing only mean values. Making use of the assumptions of the regularity of mean values in a usual way, we arrive at the following set of differential equations

$$\frac{\partial}{\partial t}\bar{\varrho} = -(\bar{\varrho}\bar{u}_j)_{,j}, \tag{3.7}$$

$$\frac{\partial}{\partial t}(\bar{\varrho}\bar{u}_i) = \bar{T}_{ij,j} - [\bar{\varrho}(\bar{\varkappa}_{ij} + \bar{u}_i\bar{u}_j)]_{,j}, \tag{3.8}$$

$$\bar{T}_{ik} = \bar{T}_{ki}, \tag{3.9}$$

$$\frac{\partial}{\partial t}[\bar{\varrho}(\bar{\varkappa}_{ik} + \bar{u}_i\bar{u}_k)] = (\bar{T}_{ij}\bar{u}_k + \bar{T}_{kj}\bar{u}_i)_{,j} + (\overline{T'_{ij}u'_k} + \overline{T'_{kj}u'_i})_{,j} - [\bar{\varrho}(\bar{u}_i\bar{u}_k\bar{u}_j + \bar{u}_i\bar{\varkappa}_{kj} + \bar{u}_k\bar{\varkappa}_{ij} + \bar{u}_j\bar{\varkappa}_{ik} + \bar{\beta}_{ikj})]_{,j} - \bar{\Pi}_{ik}. \tag{3.10}$$

Equations (3.7) to (3.10) are essentially the same as in the case of an incompressible liquid (see [1]). The form of Eqs. (3.7) to (3.10) is not quite convenient for further discussion.

Let us introduce the concept of quasi-material derivative, a counterpart of the widely used concept of material derivative. We define the quasi-material derivative denoted by an asterix as the convective derivative relative to the mean velocity field, e.g.

$$\overset{*}{\bar{\varrho}} \overset{\text{df}}{=} \frac{\partial\bar{\varrho}}{\partial t} + \bar{\varrho}_{,i}\bar{u}_i. \tag{3.11}$$

Using this concept we can rewrite Eqs. (3.7) to (3.10) in the following form:

$$\overset{*}{\bar{\varrho}} = -\bar{\varrho}\bar{u}_{j,j}, \tag{3.12}$$

$$\bar{\varrho}\overset{*}{\bar{u}}_i = \bar{T}_{ij,j} - (\bar{\varrho}\bar{\varkappa}_{ij})_{,j}, \tag{3.13}$$

$$\bar{T}_{ij} = \bar{T}_{ji}, \tag{3.14}$$

$$\bar{\varrho}\overset{*}{\bar{\varkappa}}_{ik} = (\bar{T}_{ij}\bar{u}_{k,j} + \bar{T}_{kj}\bar{u}_{i,j}) - \bar{\Pi}_{ik} + (\overline{T'_{ij}u'_k} + \overline{T'_{kj}u'_i} - \bar{\varrho}\bar{\beta}_{ikj})_{,j} - \bar{\varrho}(\bar{u}_{i,j}\bar{\varkappa}_{kj} + \bar{u}_{k,j}\bar{\varkappa}_{ij}). \tag{3.15}$$

Now it is easily seen that the only difference between the usual form of the set of equations of continuous medium and our set consists in the presence of the term $\bar{\varrho}\bar{\varkappa}_{ij}$. If $\bar{\varkappa}_{ij}$ is equal to zero, then we obtain the classical case. Equation (3.15) can be treated as the equation of evolution for an additional tensorial internal variable $\bar{\varkappa}_{ik}$. For completness of the theory we need a constitutive equation for $\bar{T}_{ij}$, $\bar{\Pi}_{ij}$ and for the third-order tensor $\bar{\Omega}_{ijk}$ defined as follows:

$$\bar{\Omega}_{ikj} = \bar{\Omega}_{kij} \overset{\text{df}}{=} \overline{T'_{ij}u'_k} + \overline{T'_{kj}u'_i} - \bar{\varrho}\bar{\beta}_{ikj}. \tag{3.16}$$

4. Discussion

The choice of quantities which have to be determined from the constitutive equations is of course, to some extent, a matter of convenience, however, if we rewrite our set of equations in integral form, for quasi-material regions, i.e. for such regions which can be considered as material region in mean (for which $U_n - \bar{\mathbf{u}}\cdot\mathbf{n} = 0$ at every point of the boundary) (³), our choice of the terms which should be determined from the constitutive equation would look more natural.

Rewriting Eqs. (3.12), (3.13) and (3.15), we obtain

$$\frac{d}{dt}\int_V \bar{\varrho}\, dV = 0, \tag{4.1}$$

$$\frac{d}{dt}\int_V \bar{\varrho}\bar{u}_i\, dV = \int_{\partial V} \bar{T}_{ij} n_j\, dS - \int_{\partial V} \bar{\varrho}\bar{\varkappa}_{ij} n_j\, dS, \tag{4.2}$$

$$\frac{d}{dt}\frac{1}{2}\int \bar{\varrho}\bar{\varkappa}_{ik}\, dV = \frac{1}{2}\int_{\partial V} (\overline{T'_{ij}u'_k} + \overline{T'_{kj}u'_i} - \bar{\varrho}\bar{\beta}_{ikj}) n_j\, dS + \frac{1}{2}\int_V (\bar{T}_{ij}\bar{u}_{k,j} + \bar{T}_{kj}\bar{u}_{i,j} - \bar{\Pi}_{ik})\, dV - \frac{1}{2}\int_V \bar{\varrho}(\bar{\varkappa}_{ij}\bar{u}_{k,j} + \bar{\varkappa}_{kj}\bar{u}_{ij})\, dV. \tag{4.3}$$ (⁴)

Multiplying Eq. (3.13) by $1/2\ \bar{u}_k$ and integrating we can also obtain

$$\frac{d}{dt}\frac{1}{2}\int_V \bar{\varrho}\bar{u}_i\bar{u}_k\, dV = \frac{1}{2}\int_{\partial V} [(\bar{T}_{ij}\bar{u}_k + \bar{T}_{kj}\bar{u}_i) - \bar{\varrho}(\bar{\varkappa}_{ij}\bar{u}_k + \bar{\varkappa}_{kj}\bar{u}_i)] n_j\, dS - \frac{1}{2}\int_V (\bar{T}_{ij}\bar{u}_{k,j} + \bar{T}_{kj}\bar{u}_{i,j})\, dV + \frac{1}{2}\int_V \bar{\varrho}(\bar{\varkappa}_{ij}\bar{u}_{kj} + \bar{\varkappa}_{k,j}\bar{u}_{i,j})\, dV. \tag{4.4}$$

From the other side, using the standard averaging procedures and making use of Eqs. (2.9) and (2.10) we can easily obtain the following expression for the mean volumetric density of kinetic energy:

$$\bar{E}^{(k)} = \frac{\bar{\varrho}}{2}(\bar{u}_i\bar{u}_i + \varkappa_{ii}), \tag{4.5}$$

i.e. we can see that the mean energy can be subdivided into two parts, the first-due to the mean velocity and second-due to the irregular movement of grains.

Denoting the mass density of energy $\bar{E}^{(k)}/\bar{\varrho}$ by $\bar{e}^{(k)}$ we can see that the traces of Eqs. (4.3) and (4.4) represent the balances of the energy connected with the irregular "microscale" movement and with the "macroscale" mean movement. Adding the traces of Eqs. (4.3) and (4.4) we can obtain the balance law for the total energy density

$$\frac{d}{dt}\int_V \bar{\varrho}\bar{e}^{(k)}\, dV = \int_{\partial V} (\bar{T}_{kj}\bar{u}_k + \overline{T'_{kj}u'_k}) n_j\, dS - \int_{\partial V} \bar{\varrho}\left(\bar{\varkappa}_{kj}\bar{u}_k + \frac{1}{2}\bar{\beta}_{kkj}\right) n_j\, dS - \frac{1}{2}\int_V \bar{\Pi}_{ii}\, dV. \tag{4.6}$$

Equations (4.3) and (4.4) contain not only information on energy, but can also be considered as the balance equations for the Euler tensor $\bar{u}_i\bar{u}_k$ and for $\bar{\varkappa}_{ij}$ — that is its counterpart on the "micro" level. We can see, for example, that some terms cancel each other when we add Eqs. (4.3) and (4.4), namely, the term

$$(\bar{T}_{ij}\bar{u}_{kj} + \bar{T}_{kj}\bar{u}_{ij}) - \bar{\varrho}(\bar{\varkappa}_{ij}\bar{u}_{k,j} + \bar{\varkappa}_{kj}\bar{u}_{i,j})$$

(³) On the correctness of such a concept see [6].

(⁴) We multiplied Eq. (4.3) by 1/2 for the purpose of further considerations.

appears in both equations with the opposite sign; this makes it possible to consider this term as the local rate of exchange between the microscale and macroscale motion ([5]). The surface terms represent the energy and momentum exchange due to the stress, and due to the transport of moving grains across the boundaries of the region (we recall here that the boundaries of such regions can be considered as the boundaries of material region only in mean). These arguments suggest that we can consider these terms as fluxes, i.e. that we have to look for the constitutive equations for $\bar{\Omega}_{ikj}$ rather than for $\bar{\Omega}_{ikj,j}$. It should be mentioned here we will not develop, however, this line in the present paper that we can point out some analogy between $\bar{\varkappa}_{ij}$ or its trace and the temperature of the gas; in this case the third-order tensor $\bar{\Omega}_{ikj}$ can be considered as a tensorial counterpart of the heat flux vector.

5. Final remarks

According to the author's opinion the validity and the practical applicability of the present considerations as well as the form of constitutive relations could be verified only by experiments; however, it is quite possible that some further information can be obtained through consideration on the level of individual grains in contact with their neighbours. It is also quite possible that the subdivision of the irregular rigid grain movement into rigid rotation and progressive motion can also be fruitful.

Lastly, it may be noted that even in its present form the model under consideration can provide some information on the possible form of the constitutive relations, namely,

a) the matrix $\bar{\varkappa}_{ij}$ should be positively defined;

b) $\bar{Д}_{ik} = \bar{Д}_{ki}$;

c) $\bar{\Omega}_{ikj}$ should be symmetric in two first indices and cannot be obtained as an isotropic function of $\bar{\varkappa}_{ij}$, $\bar{\varrho}$ and $\bar{u}_{(i,j)}$ only, i.e. if it appeared from experiments that $\bar{\Omega}_{ikj}$ should be taken different from zero, then it should depend on the gradient of at least one among the quantities $\bar{\varkappa}_{ij}$, $\bar{\varrho}$ and $\bar{u}_{(i,j)}$;

d) we can expect that $\bar{Д}_{ij}$ or $\bar{Д}^*_{ij}$ is sensitive to the deviatoric part of $\bar{\varkappa}_{ij}$ expressing the tendency of $\bar{\varkappa}_{ij}$ not only to decay in the absence of the gradient of the mean velocity, but also to become more spherical.

The problem of the boundary conditions is not discussed here and remains an open question for further investigation.

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([5]) Here also another possible interpretation arises: we can consider separately $\bar{T}_{ij}\bar{u}_{k,j}+\bar{T}_{kj}\bar{u}_{i,j}$ and the term $\bar{T}_{ij}\bar{u}_{k,j}+\bar{T}_{kj}\bar{u}_{i,j}-\bar{Д}_{ik}$ denoting the last, e.g. by $\bar{Д}^*_{ik}$ and considering it as a dispersion on "micro" level.

References

1. А. С. Монин, А. М. Яглом, *Статистическая гидромеханика*, ч. I, Наука, Москва 1965.
2. Б. И. Давыдов, *К статистической динамике несжимаемой турбулентной жидкости*, Доклады Академии наук, 127, 4, 768–772, 1959.
3. Б. И. Давыдов, *К статистической теории турбулентности*, Доклады Академии наук, 127, 5, 980–983, 1959.
4. Б. И. Давыдов, *К статистической динамике несжимаемой турбулентной жидкости*, Доклады Академии наук, 136, 1, 47–51, 1961.
5. Б. И. Давыдов, *Феноменологические уравнения статистической динамики несжимаемой турбулентной жидкости*, Журн. эксп. теор. физ., 35, 2, 527–529, 1958.
6. A. Blinowski, *Gradient description of capillary phenomena in multicomponent fluids*, Arch. Mech., **27**, 2, 273–292, 1975.

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